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Maintenance Model for an Automobile System

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***Abstract*—**The maintenance model for a repairable system is considered. The system will fail due to an internal component failure, or an external arrival of an accident, whichever occurs first. The consecutive threshold values of an external arrival of an accident are monotone geometric. Whenever the inter-arrival time of two successive small repairable breakdowns is smaller than the specified threshold then the system fails and will be replaced. The optimal policy N^* for minimizing the long run average cost per unit time is determined with a numerical example.

***Keywords*—**External Cause, Internal Cause, Maintenance Model, Poisson Process, Shock Models, Terminal Small Repairable Breakdown

***Abbreviation*—**Geometric Process (GP)

I. INTRODUCTION

CONSIDER an automobile system will fail due to an internal cause (component failure) or due to an external arrival of the terminal small repairable breakdown (accident) whichever occurs first. Suppose the automobile system fails due to an external arrival of an accident, whenever the inter-arrival time of two successive small repairable breakdowns is smaller than the specified threshold then the system fails. Because the GP model is a good model for a system that fails due to its internal failure and the δ-shock model is a reasonable model for a system that fails due to an external failure. A GP, δ-shock maintenance model as a combination of the above two failure models for a repairable system. A lot of research has been done in the theory of the GP, and its application to the maintenance problem. Barlow & Prochan (1975) studied the shock model. That describes, whenever a shock arrives, it will cause a random amount of damage to the system. A shock is a deadly shock, when the accumulated amount of damage to the system by the arrival time of the shock exceeds a threshold, and then the system fails. A number of papers on the maintenance of the system subject to shocks were published by Gottlieb (1982), Shanthikumar & Sumita (1983) and others. In Shanthikumar & Sumita (1983), system failure in shock model approach considered, and a sufficient conditions evaluated under which the system failure time is completely monotone. In Gottlieb (1982), the general case analyzed where the failure rate need not be increasing and replacement can be made at any time. Also optimal replacement policy is found and conditions for optimal policies are determined.

Lam (1988 A), first introduced the Geometric process and its application to maintenance model and in Lam (1988), discussed the monotonicity optimal replacement policy using GP models. In Lam (1991), he discussed the successive operating times of a system decreasing while the consecutive repair times after failure will be increasing for a deteriorating models. Wu & Clements - Croome (2005) developed the maintenance policies for the system under study operates iteratively at two successive states: up or down. The costs due to system failure at the up state consist of both business losses & maintenance costs, whereas those at the down state only include maintenance costs. Braun et al., (2005) investigated the delta-shock model of complex systems reducing in to a linear combination of parallel systems. Zhi Sheng Ye et al., (2011) proposed the degradation and shock are two common mechanisms accounting for product failures. In a single model when the extent of degradation and the magnitude of shocks are not observable, but only the failure times and the corresponding failure modes are recorded. Lam & Zhang (2003) studied a δ-shock maintenance model by assuming that a system can work forever when there is no terminal δ-shock. A δ-shock model was considered by Zehui Li & Peng Zhao (2007), and Bai et al., (2006). Lam & Zhang (2004), in a refreshing departure introduced a new class of shock models and called them δ–shock models. While the earlier shock models concerned solely on the magnitude of the damage caused by the shocks, Lam (2007) introduced monotone process model and the threshold geometric process model is used to study data sets of daily infected case in SARS (Severe Acute Respiratory Syndrome). Tang & Lam (2006) model paid attention to the frequency of the shocks. Yeh Lam (2013), recently studied the monotonicity properties of the optimal replacement policy for an improving system, proved that the optimal replacement policy is the 8 policy, i.e., the policy without replacement.

In practice, the operating time of a system is a random variable because the system can fail due to an external arrival of an accident .In this paper a GP maintenance model is studied. The optimal policy N^* for minimizing the long run average cost per unit time is determined. When there exists no shock, the successive operating time of the system after repair forms GP. The shocks will arrive according to Poisson process. The frequency of breakdowns rather than the accumulated amount of damage of breakdowns. Whenever the inter arrival time of two successive small repairable breakdowns is smaller than a specified threshold the system fails, and later shock is called a terminal small repairable breakdown. Thus in a δ – shock model, a shock is a terminal small repairable breakdown if the time between two breakdowns is less than a pre-specified value δ (the threshold value), and the system fails at the time of the occurrence of the terminal small repairable breakdown. In this model an external arrival of a terminal small repairable breakdown considered and optimal policy determined to minimize the Long run average cost.

II. NOTATIONS

ω – Rate of Poisson process of the arrival of shocks
λ – Changing rate of a terminal small repairable breakdown
δ – Threshold of a terminal repair of a new system
γ – Rate of Poisson process of the arrival breakdown
α – Constant ratio of GP for the successive operating time
y – Constant ratio of GP for the consecutive repair time
z – The repair cost has a constant rate
r – Reward rate, R – The replacement cost
a – The replacement time, μ_p – Constant rate
P_n – The operating time of a system following $(n-1)^{th}$ failure
Q_n – The repair time of the system after n^{th} failure
N – A policy by which the system will be replaced following the N^{th} failure.
N^* – Minimizing the long run average cost
C(N) – Long run expected cost, g(N) – An auxiliary function
$\theta - E(P_{11})$, $\theta_n - E(P_n)$, $\xi - E(Q_n)$, $\Psi - E(a)$

III. ASSUMPTION OF THE MODEL

1. At the beginning, a new system is new whenever the system fails it will be repaired. The system will be replaced by a new, identical one following the N^{th} failure.
2. A system subject to attacks from sequence of repairs. The repair will arrive according to a Poisson process with rate ω .
3. If a system has been repaired as n times: n = 0,1,2,… The threshold of a terminal small repairable breakdown will be $\lambda^n\delta$. If there exists no repair.
4. Let P_n be the operating time of a system following $(n-1)^{th}$ failure. Assuming that the operating time $\{P_n;\ n = 1,2,3,\ldots\}$for a GP with ratio x and P_n has Weibull distribution with density function
$$f_n(x) = \begin{cases} \dfrac{(\alpha\beta)^{n-1}}{\theta} x^{\beta^{n-1}-1} \exp(-(\alpha\beta)^{n-1} x^{\beta^{n-1}-1} / \theta) & x > 0 \\ 0 & \textit{other wise.} \end{cases} \qquad (1)$$
$$\textit{where}\ \ \alpha, \beta > 0$$
The system will fail at the end of the running time or arrival of the terminal small repairable breakdown (accidents) which ever occur first.
5. Let Q_n be the repair time of the system after n^{th} failure. When the repair time $\{Q_n\}$ constitute a GP with ratio y. The mean repair time following the first failure is $E(Q_n) = \xi \geq 0$ .During the repair time the system is ideal. A replacement time is a random variable „a‟ with $E(a) = \psi$.
6. The repair cost has a constant rate „z‟, and the reward when the system is operating and constant rate „r‟. The replacement cost „R‟, and a cost proportional to the replacement time „a‟ at a constant rate μ_p.
7. The Geometric process and the Poisson process are independent.
8. $\lambda \geq 1$, $\alpha, \beta \geq 1$, and $0 < y \leq 1$.

IV. LONG RUN AVERAGE COST

A cycle is a time interval between the installation of a system and the first replacement, or a time interval between two consecutive replacements. Therefore, the successive cycles will form renewal process. Consequently, the successive cycles together with the cost incurred in each cycle will constitute a renewal processes [Shanthikumar & Sumita, 1983].

The long-run average cost per unit time is calculated using the following formula,

$$\frac{\textit{Expected cost incurred in a cycle}}{\textit{Expected length of a cycle}} \qquad (2)$$

Moreover, suppose a replacement policy N is adopted. Let the Long run average cost be denoted by C(N).

$$C(N) = \frac{z \sum_{n=1}^{N-1} E(Q_n) - r \sum_{n=1}^{N} E(P_n) + R + \mu_p a}{\sum_{n=1}^{N} E(P_n) + \sum_{n=1}^{N-1} E(Q_n) + a} \qquad (3)$$

Let P_n be the real operating time of the system following $(n-1)^{th}$ repair in a cycle, and its operating time,

$$\theta_n = E(P_n) = \frac{1}{\gamma + (\alpha\beta)^{n-1}/\theta - \gamma \exp(-(\gamma + (\alpha\beta)^{n-1}/\theta)\tau^{n-1}\delta)} \qquad (4)$$

Assume $\gamma = 0$, that there is no shock .so the system will fail due to the arrival of an external terminal small repairable breakdown. Consequently, (4) yields,

$$\theta_n = E(P_n) = \frac{\theta}{(\alpha\beta)^{n-1}} \tag{5}$$

Thus, the GP δ shock model reduces to the GP model. Then,

$$C(N) = \frac{z\xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} - r \sum_{n=1}^{N} \frac{\theta}{(\alpha\beta)^{n-1}} + R + \mu_p \psi}{\sum_{n=1}^{N} \frac{\theta}{(\alpha\beta)^{n-1}} + \xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \psi} \tag{6}$$

To determine an optimal policy N^* for minimizing the average cost, rewrite (6) as C(N) = A(N) – r, Where,

$$A(N) = \frac{(z+r)\xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + R + (\mu_p + r)\psi}{\sum_{n=1}^{N} \frac{\theta}{(\alpha\beta)^{n-1}} + \xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \psi} \tag{7}$$

Then, minimize C(N) is equivalent to minimize A(N), and A(N+1) – A(N) will always positive. The auxiliary function for minimizing A(N). i.e., A(N+1)>A(N)⇔g(N)>1.

$$g(N) = \frac{(z+r)\xi\left(\sum_{n=1}^{N} \frac{\theta}{(\alpha\beta)^{n-1}} - \theta_{N+1} \sum_{n=1}^{N-1} y^n + \psi\right)}{(R + (\mu_p + r)\psi)(\theta_{N+1} y^{N-1} + \psi)} \tag{8}$$

V. NUMERICAL EXAMPLE

Consider the parameter values be τ=1.2, α=1.07, β=1, y=0.97, θ=30, ξ=20, ψ=10, δ=2, γ=0.07, z=25, r=100, μ_p=10 and R=3500.

Table 1 – Values of C(N) and g(N)

N	C(N)	g(N)	N	C(N)	g(N)
1	37.88	0.0197	6	-17.888	1.0654
2	-0.035	0.0653	7	-16.657	1.2653
3	-11.390	0.2543	8	-15.867	1.5634
4	-15.697	0.6443	9	-13.453	1.8347
5	**-18.886**	**1.0186**	10	-12.180	2.1098

VI. CONCLUSION

From the above table, it is clear that C(5) = -18.886 is the unique minimum average cost. Because C(N) is decreasing when N < 5,and increasing when N>5. Therefore, an optimal replacement Policy is N^* =5.and also, it is easy to see that 5 is the first integer so that g (N) > 1.i.e. g (5) = 1.0186 > 1. This means that the optimal policy to replace the system at the time following the 5th failure. Thus the GP shock model as a maintenance model for an Automobile system that takes into account the effect of an internal cause, or an external cause on the auto mobile system. This will effectively determine the proper time to replace the system wherever equipment, machinery parts available.

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