Analysis and Mathematical Modelling of HIV/AIDS and Transmission Dynamics subject to the Influence of Public Health Education Campaign

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***Abstract*—**In this study, we have analyzed a nonlinear mathematical model for the spread of HIV/AIDS in a population with variable size structure. A threshold parameter, basic reproduction number, , was found that completely determined the stability dynamics and outcome of the disease. It was found that when, 1, the disease free equilibrium is stable and the disease dies out. However, when, 1, there exists a unique endemic equilibrium that is locally asymptotically stable. It was concluded from our analysis that endemic level of infective population increases with the increase in rate of transmission of infection due to infective among susceptible class. This study presented a nonlinear extended deterministic Susceptible Infected model for assessing the impact of public health education campaign on curtailing the spread of the HIV pandemic in the Kenyan population. Rigorous qualitative analysis of the model were used to compare between the model without education and that with education in order to determine the stability of disease-free equilibrium with a stable endemic equilibrium when the basic reproduction number is less than unity. The model was used to assess the potential impact of some targeted public health education campaigns using data from the 47 Kenyan counties. Numerical simulation of the model is also performed by using fourth order Runge - Kutta method. The results from the model suggested that the most effective prevention measure that would be applied is by public health education campaign in order to thwart the epidemic. As expected, the control strategy would be more effective if all interventions were implemented together.

***Keywords*—**Basic Reproduction Ratio; Disease-Free Equilibrium; Education; HIV/AIDS Dynamics; Steady State Equilibrium.

***Abbreviations*—**Acquired Immuno Deficiency Syndrome (AIDS); Antiretroviral (ARV); Human Immunodeficiency Virus (HIV); Joint United Nations Programme on HIV/AIDS (UNAIDS); World Health Organization (WHO).

I. INTRODUCTION

EPIDEMIOLOGY is the study of the distribution and determinants of diseases, for both infectious and non-infectious diseases. Originally the term was used to refer only to the study of epidemic infection diseases, but it is now applied more broadly to other diseases as well. Mathematical models have become important tools in analyzing the spread and control of infectious diseases. The model formulation clarifies assumptions, variables and parameters. Moreover models provide conceptual results such as thresholds, basic reproduction numbers, contact numbers and replacement numbers. Understanding the transmission characteristics of infectious diseases in communities, regions and countries can lead to better approaches to decreasing the transmission of these diseases. Mathematical models are used in comparing, planning, implementing, evaluating and optimizing various detection, prevention, therapy and control programs. HIV/AIDS is a serious and highly infectious disease which may lead to hospitalization or death. Over 1,000 children are newly infected with HIV every day, and of these more than half will die as a result of AIDS because of a lack of access to HIV treatment. At the end of 2010, there were 3.4 million children living with HIV around the world. In addition to this, millions more children every year are indirectly affected by the epidemic as a result of the death and suffering caused in their families and communities. Record also has it that over 390,000 children became newly infected with HIV in 2010. It is also in record that 1.8 million people who died of AIDS during 2010, one in seven were children. Every hour, around 30 children die as a result of AIDS.

Most children living with HIV/AIDS are from sub-Saharan Africa where AIDS is known to have its greatest toll in the world. While HIV/AIDS can be considered a global pandemic, it is by any account overwhelmingly an African one. In Africa, HIV is transmitted mostly through sexual intercourse. Currently, despite some promising trends public health education in most countries, HIV/AIDS epidemic continues to grow tremendously. Epidemiology is the study of the distribution and determinants of diseases, for both infectious and non-infectious diseases. Originally, the term was used to refer only to the study of epidemic infection diseases, but it is now applied more broadly to other diseases as well. Mathematical models have become important tools in analyzing the spread and control of infectious diseases.

HIV levels in the bloodstream are typically highest when a person is first infected and again in the late stages of the illness. The progression of HIV infection to AIDS probably depends on how well the body can replace cells destroyed by virus. Modes of HIV transmission Epidemiological evidence shows that HIV is transmitted only through the intimate exchange of body fluids, such as blood, semen, vaginal secretion, and mother's milk. Thus, HIV could be passed from an infected mother to her child (i.e., vertical infection) during pregnancy, birth or through infected breast milk.

1.1. Statement of the Problem

Despite the availability of the HIV/AIDS drugs (oral PREP, oral PEP and ARVs) the infectious disease is still endemic in many parts of the world including developed nations. The disease has continued to cause both economic and health problems to large populations worldwide mostly affecting the energetic sexually active group. Due to these impacts, this study aims to develop a mathematical model for control and elimination of the transmission dynamics of HIV/AIDS. For most people, infection with HIV is the start of a progressive illness that results in death due to the increasing occurrence of opportunistic infections. The spread of the HIV pandemic in a population continues to be a major public health problem more specifically in developing countries like Kenya. The emergence of drug-resistant HIV strain in the last three decades has been a major problem in tackling this scourge. A mathematical model for investigating the impact of that role of public health education campaign on the transmission dynamics of HIV/AIDS is developed.

1.2. Motivation of this Research

The motivations of this research are to:

a) Give a detailed explanation on the understanding of the transmission of HIV/AIDS between the educated population and the uneducated population for the control of HIV/AIDS in Kenya and other developing countries.
b) Perform simulation of the mathematical model which shall help to assess various biological factors that are affecting the spread of HIV/AIDS
c) Find the best practical and working control and elimination of the transmission dynamics of HIV/AIDS.
d) To extend some of the aforementioned studies, by designing and analyzing a new comprehensive model, for HIV transmission in a population, that incorporates the role of public health education campaign and using the model to evaluate the impact of some targeted public health education strategies and this shall indeed reduce the unfilled gap from the early researchers.

1.3. Research Objectives

1.3.1. General Objectives

To model mathematically the progress of an infectious disease in order to discover the likely outcome of an epidemic.

1.3.2. Specific Objectives

The objectives of this research are to:

a) Formulate a mathematical model for the HIV/AIDS on transmission dynamics.
b) To obtain the Disease Free Equilibrium (DFE) and endemic equilibrium (EEP) points for HIV/AIDS disease in Kenya.
c) To determine the basic reproduction number and its qualitative analysis for HIV/AIDS in Kenya.

1.4. Significances of this Research

a) The government and heath organizations can use findings of this research to plan for the best control programmes like use of oral PREP, oral PEP and ARVs and hence prevent future HIV/AIDS pandemic.
b) To enable the public to participate in various prevention and protection programmes since they shall be aware of how it is best to protect future HIV/AIDS infections.
c) The analysis of the dynamics of HIV/AIDS transmission can be used to predict HIV/AIDS likely spread before it occurs.
d) This research will contribute to improve future studies of HIV/AIDS mathematical modeling.

1.5. Assumptions of the Model

(i) The AIDS patients who received public health education die due to AIDS at a slower rate than the AIDS patients who did not.

(ii) Public health education will be offered to all infected individuals except for the education of high-risk people with AIDS and hence will not only be restricted to susceptible individuals.

(iii) Intensive public health education of newly-recruited sexually-active individuals will be carried out.

II. Related Works/Literature Survey

In this section we review in brief mathematical models of HIV/AIDS developed previously. Mathematical models have a long history of being used by different governments and organisations in the world in controlling and elimination strategies of infectious diseases. The following are some of the mathematical models developed to study transmission dynamics of HIV/AIDS using different approaches;

HIV is classified as an infectious disease which rapidly spreads amongst communities and changes its distributions in space, time and social space [Elbasha & Gumel, 7]. Many factors, including increased mobility, are associated with an increased risk of HIV infection [Chamuchi Moffat Nyaboe et al., 4]. The transmission of HIV is also strongly associated with the spatial distribution of high risk groups. The distribution of AIDS cases not only varies by cities and states, but also by geographical regions, [Altmann, 1]. The models that are labeled by SI, SIS, SEIS, and SEIR are mostly used where the sub-populations are Susceptible, Exposed, Infected and Recovered or Removed.

Chamuchi Moffat Nyaboe et al., [5] looked at a model of HIV/AIDS that examine the diminution in infection by promoting a change in sexual behavior through public health information campaigns and individuals with AIDS to abstain from sexual activities. Both the endemic and disease free equilibrium have been investigated. Numerical simulations are presented using the fourth order of Runge-Kutta. The results from their research have shown that media campaigns had led to a reduction in the prevalence of the disease but may not be the only ultimate strategy in the fight against HIV/AIDS. It has also shown that an increase in the distribution of public health information campaigns has led to a decrease in occurrence of a disease. In the case of the individual with AIDS abstaining from sexual activities, it has also reduced the effect of the disease.

The impact of educational campaigns as a control measure for the spread of HIV/AIDS has been investigated by LaSalle [10]. The authors present a sexual transmission model with explicit incubation period. Their results suggested that educating sexually immature and sexually mature individuals concurrently is more effective in slowing down HIV/AIDS than concentrating on cohort public health educational campaign of sexually immature or sexually mature individuals only. It is shown that in their study, in situations where education is effective and with reasonable average number of HIV infected partners, public health campaigns can slow down the epidemic.

An epidemic HIV/AIDS model with treatment has been investigated in the research paper by Nyabadza & Mukandavire [8]. The model allows some infected individuals to move from symptomatic phase to the asymptomatic phase by all kinds of treatments. The authors introduced the time delay to the model in order to investigate the effect of the time delay on the stability of the endemically infected equilibrium. This discrete time delay has also been used to the model to describe the time from the start of the treatment in the symptomatic stage until the treatment effects becomes clear. It was found that treatment can be used to make the disease free equilibrium (E_0) stable when it would be unstable in the absence of treatment. On the other hand using the time delay can induce oscillation in the system. Biologically, this means that there is a critical value for the treatment-induced delay which determines the stability of the infected equilibrium. That is, the infected equilibrium is asymptotically stable when antiretroviral drugs on average show positive effects in patients within less than time delay.

A continuous model for HIV/AIDS disease progression has been formulated and physiological interpretations were provided by May & Anderson [11]. The abstract theory was then applied to show existence of unique solutions to the continuous model describing the behavior of the HIV virus in the human body and its reaction to treatment by antiretroviral therapy. The product formula has suggested appropriate discrete models describing the dynamics of host pathogen interactions with HIV1 and is applied to perform numerical simulations based on the model of the HIV infection process and disease progression. Finally, the results of the numerical simulations are visualized and it was observed that they agreed with medical and physiological aspects.

A simple deterministic HIV/AIDS model incorporating condom use, sexual partner acquisition, behavior change and treatment as HIV/AIDS control strategies has been formulated by Wang [15] using a system of ordinary differential equations with the object of applying it to the current South African situation. The authors fit the model to a data from [14], in South Africa and the epidemiological facts sheets shows the current prevalence scenario. The results compare very well with other research outcomes on the HIV/AIDS epidemic in South Africa. Projections were made to track the changes in the number of individuals who were able to be under treatment, an important group as far as public health planning is concerned.

Perelson & Nelson [12] formulated a deterministic HIV/AIDS model that incorporates condom use, screening through HIV counseling and testing (HCT). A regular testing and treatment as control strategies has been proposed with the objective of quantifying the effectiveness of HCT in preventing new infections and predicting the long-term dynamics of the epidemic. The authors fit the model to a current prevalence data in South Africa from Daniel [6] reports and epidemiological fact sheet. They looked at a recently launched HTC campaign to model its possible impact on the dynamics of the disease. The model shows that HTC alone has a very little impact in reducing the prevalence of HIV unless the ability of the campaign exceeds an evaluated threshold in the absence of bifurcation. The result has shown that force of infection can only be reduced through behavior change, condom use and reduction in the number of sexual partners and these form the pillars of prevention of new infection. The results have shown that the presence of bifurcation has an important implication in the control of HIV/AIDS. The model has shown that it cannot be eliminated by simply reducing the value of reproduction number R_0 to below unity.

Chavez & Song [3] have considered a more robust systematic and complete qualitative analysis of a two strain HIV/AIDS model with treatment of AIDS patients. The treatment with amelioration results in an increase in number of HIV patient and a decrease in Aids patients. They have advised that treatment with amelioration should always be accompanied by public health education. The authors investigated that if the drugs used for therapy are 100 percent effective and a positive change in the sexual behavior of treated individuals is achieved, treatment with amelioration will not increase the development of HIV/AIDS in societies but will help communities by lengthening the lives of the infected, thus, reducing morbidity/mortality and socio-economic costs. Further the analysis of the reproduction numbers show that the use of antiretroviral therapy to improve the quality of life of AIDS patients with antiretroviral sensitive, HIV results in an increase of antiretroviral resistant HIV cases supporting the argument that antiretroviral resistance develops as a result of selective pressure on non-resistant strains due to antiretroviral use, [Chamuchi Moffat Nyaboe et al., 4].

A non-linear mathematical model has been proposed and analyzed to study the spread of HIV/AIDS with direct inflow of infective in a population with inconsistent volume structure. Mukandavire & Garira [17] has looked at a model without inflow of HIV infective including interaction with pre-AIDS individuals and Model without inflow of HIV infective and no interaction with pre-AIDS individuals. It was found that if the direct inflow of the infective has been allowed in the community the disease always persist. The endemicity is extensively reduced if direct inflow of infective is restricted and pre-Aids individuals do not take place in sexual activities. Sharomi et al., [13] suggested sexual partners should be restricted and unsafe sexual iteration should be avoided with an infective in order to reduce the spread of the disease. Thus the spread of infection can be slowed down if direct inflow of infectives is restricted into the population. It was also noted that the increase in the number of sexual partners further reduces the total population by way of spreading the disease. Thus in order to reduce the spread of the disease, the number of sexual partners should be restricted and unsafe sexual interaction should be avoided with an infective.

Garba & Gumel [9] has proposed the interaction of the immune system and human immunodeficiency virus where we will introduce the possibility of using highly active anti-retroviral therapy (HAART) to stimulate the vaccine. They further present a Model Predictive Control (MPC) based method for determining optimal treatment. Finally they analyze the simulations by using algorithms where they apply robustness measurement noise, robustness modeling error, robustness combined errors, and varying the cost function. An SIR model with six compartments where there is an interaction between HIV and TB epidemics has been investigated. They further look at sensitivity of the steady states with respect to changes in parameter values. The authors examine that most of the control measures studied have an obvious positive impact in controlling the HIV or TB epidemics; this is the case for condom use, increased TB detection and preventive treatment. The situation for ART is more complicated. However, although the future for the prevalence of HIV is uncertain, it seems that a generalized access to ART would lead to a significant decrease of the TB notification rate. They further concluded that it is difficult to guess if the observations drawn from the model with parameters adapted to the particular South African township are still valid for less crowded areas with high HIV prevalence, finally reliable data on both HIV and TB are still rare.

Yan & Zou [16] formulated and analyzed a sex-structured model for heterosexual transmission of HIV/AIDS. The model has been further divided into two classes, consisting of individuals involved in high-risk sexual activities and individuals involved in low-risk sexual activities. The model is described as the movement of individuals from high to low sexual activity group as a result of public health education campaigns. The threshold parameter which is the basic reproduction number has been obtained and their stability (local and global) of the disease free equilibrium. The model has been extended to incorporate sex workers, and their role in the spread of HIV/AIDS in settings with heterosexual transmission was explored. In order to assess the possible community benefits of public health educational campaigns in controlling HIV/AIDS comprehensive analytic and numerical techniques were employed.

Standard mathematical models of the spread of infectious diseases are well known and have been widely applied for many diseases including HIV in different regions in the world [Brauer & Chavez, 2]. There is still no cure or vaccine for HIV, and anti-retroviral drugs (ARVs) are still not widely accessible, particularly in the resource-poor nations (which suffer the vast majority of the HIV burden globally). Yet, HIV remains preventable through the avoidance of high-risk behaviors, such as unprotected sexual intercourse and sharing of drug injection needles.

Public health education campaigns have been successfully implemented in numerous countries and communities, such as: Uganda, Thailand, Zambia and the US gay community, [Elbasha & Gumel, 7]. Between 1991-1998, HIV prevalence dramatically declined in Uganda from 21% to 9.8% (with a corresponding reduction in non-regular sexual partners by 65% coupled with greater levels of awareness about HIV/AIDS; [Wang, 15].

III. Model Formulation and Analysis

Models are used to have constant total population in assessing the effectiveness of public health policies aimed at the control of HIV/AIDS epidemics though it may fail to capture the severity of the epidemic in populations which are undergoing demographic changes. Mathematical models have been used for centuries to develop a better understanding of systems in order to control or optimize results.

3.1. Basic Properties of the Model

Since the model monitors human population, all its associated parameters and state variables are assumed to be non-negative for all t ≥ 0. Before analyzing the model, it is instructive to show that the state variables of the model remain non-negative for all non-negative initial conditions. The following theorems help us understand and prove that our model is mathematically well defined and biologically feasible.

3.1.1. Positivity and Boundedness of Solutions

Positivity of Solutions Model describes a human population, and, therefore it is very important to prove that all quantities (susceptible, infected and those with AIDS symptoms population) will be positive for all times. Positivity implies that the system persists i.e. the population survives.

Theorem1: Let the initial values of the state variables be $(t) \geq 0$, $I(t) \geq 0$, $A(t) \geq 0$. *Then we show that, for every* 0, *the solution set* $\{\ (t), I(t), A(t)\}$ *of our model is non-negative and* Γ *is the invariant region.*

Proof: Taking our model equations we have that;

For the susceptible individuals we have,

$dS(t)$

$S(t) \geq S(0)$

Hence S is non-negative for all 0.

Similarly it can be shown that I and

3.1.2. Invariance Principle

Since general epidemiology models monitor human populations, it is necessary to consider that associated population sizes can never be negative. Thus, epidemiological models should be considered in (feasible) regions where such property (non-negative) is preserved.

Theorem 2: Suppose that the equilibrium point of system (3.10) $\bar{x} = 0$ *and V is a Liapunov function on some neighbourhood U of* $\bar{x} = 0$. *If* $x_0 \in U$ *has its forward trajectory bounded with limit points in U and M is the largest invariant set of*

$E = \{\ \bar{x} \in U : \dot{V}(\bar{x}) = 0\}$,

then $\varphi\ (x_0, t) \to M$ *as* $t \to \infty$.

Proof: A set M is an invariant set with respect to a system of ordinary differential equations.

˙ = f (x) if

x (0) ∈ M ⇒ x (t) ∈ M, for all t ∈ R.

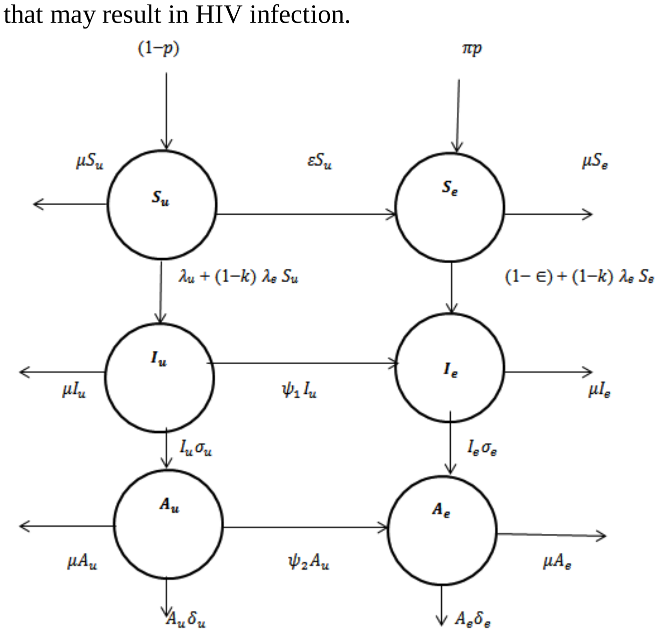
A set M is a positively invariant set with respect to

$\dot{x}$ = f (x) if

x (0) ∈ M ⇒ x (t) ∈ M, for all t ≥ 0.

3.2. Compartmental Diagram

The total population at time t, denoted by N (t), is sub-divided into the following mutually exclusive sub-populations: uneducated susceptible individuals (S_u(t)), educated susceptible individuals (S_e(t)), uneducated infected individuals with no AIDS symptoms (I_u(t)), educated infected individuals with no AIDS symptoms (I_e(t)), uneducated infected individuals with AIDS symptoms (A_u(t)) and educated infected individuals with AIDS (A_e(t)). (Un)Educated means individuals who (do not) receive proper public health education or counselling against risky practices that may result in HIV infection.



The rates ¸ λ_u and λ_e above are the forces of infection associated with HIV transmission by uneducated (at the rate $_u$) and educated (at the rate λ_e) infected individuals, respectively. The parameter β is the effective contact rate (that is, contact that may result in HIV infection), while the parameters $\eta_u > \eta_e > 1$ account for the relative infectiousness of individuals with AIDS symptoms in comparison to the corresponding infected individuals with no AIDS symptoms. Recruitment into the sexually-active population occurs at a rate π (all newly-recruited individuals are assumed to be susceptible to HIV infection),and a fraction, p, of these newly-recruited sexually-active individuals are assumed to be educated about the risks and consequences of the HIV disease. Uneducated susceptible individuals (excluding the newly-recruited individuals) receive education about safer sex practices at a rate ξ. Susceptible people acquire infection following effective contact with infected individuals (at the rates λ_u and λ_e). It is assumed that educated infected individuals (in I_u or A_u class) modify their behaviour positively, thereby reducing their risk of HIV transmission by a factor κ, with $0 < \kappa < 1$. In other words, it is assumed that HIV-infected individuals that received public health education transmit the disease at a lower rate in comparison to uneducated HIV infected individuals. Educated susceptible individuals acquire infection at a reduced rate $(1-\epsilon)[\lambda_u + (1-k)\lambda_e]$, where $0 < \epsilon < 1$ is the efficacy of public health education in preventing new infection of educated susceptible individuals. Uneducated infected individuals progress to AIDS at a rate σ_u, while educated infected individuals progress at a reduced rate $\sigma_e < \sigma_u$, (in other words, infected individuals who received public health education progress to AIDS at a slower rate in comparison to those who do not). Uneducated infected individuals without AIDS symptoms I_u are educated at a rate ψ_1, and move to the corresponding educated infected class I_e. Individuals in all classes suffer natural death at a rate μ. Additionally, individuals with AIDS die at a rate δ_u (for the uneducated class) or δ_e (for the educated class) such that $\delta_e < \delta_u$. Thus, it is assumed that AIDS patients who received public health education die due to AIDS at a slower rate than the AIDS patients who do not. Uneducated individuals with symptoms of AIDS (A_u) are educated at a rate ψ_2, and move to the corresponding educated class (A_e). It will be investigated about the manner in which the educated infected individuals (in I_u or A_u class) modify their behaviour positively in order to reduce their risk of HIV transmission by a factor k, with 1.

3.3. Differential Equations

In addition to the aforementioned extensions, this study will contribute to the literature by giving detailed qualitative analysis of the model. The model takes the form of the following deterministic system of nonlinear differential equations:

$$
\left.\begin{aligned}
&— \quad \pi(1-P) - \quad -[\quad (1-k)\lambda_e]S \\
&— \quad (1-\epsilon)[\lambda \quad (1-k)\lambda_e]S \\
&— \quad [\lambda \quad (1-k)\lambda_e]S \quad - \\
&— \\
&— \quad (1-\epsilon)[\lambda \quad (1-k)\lambda_e]S \\
&—
\end{aligned}\right\} \qquad (1)
$$

Where,

$= \frac{\beta(\lambda \quad u)}{}$ and $= \frac{\beta(\lambda \quad e)}{}$

with $N(t) \quad (t) \quad (t) \quad (t) \quad (t)+ \quad (t)+ \quad (t)$

3.4. Basic Properties of the Model

Before commencing the steady-states and analysis of the model (1), it is important to look at some properties to ensure existence of mathematically meaningful solutions.

3.4.1. Boundedness and Positivity of the Model Solutions

In this sub-section, we prove that the model system (1) is mathematically well defined and biologically feasible.

Theorem 1: Let the initial values of the state variables be $(0) \geq 0, S_e(0) \geq 0, I_u(0) \geq 0, I_e(0) \geq 0, A_u(0) \geq 0,$ $(0) \geq 0$. *Then show that, for every* $t > 0$ *the solution set* $\{ \quad (t), S_e(t), I_u(t), I_e(t) \quad (t), A_e(t)\}$ *of the model (5.10) is non-negative and* Γ *is the invariant region.*

Proof: Taking the first equation (1) for the uneducated susceptibles individuals we have,

$$— \quad \pi(\quad) \quad -[\quad (1-k)\lambda_e]S$$

$$\{- \quad -[\quad (1-k)\lambda_e]S \quad \}$$
$$\{-[\quad (1-k)\lambda \quad \mu]\}$$
$$(t) \quad (0)e^{\int\{-[\quad +(1-k)\lambda \quad \mu]\}dt}$$

Hence is non-negative for all

Similarly it can be shown that $(t) \quad (t)$ $(t) \geq 0, A_u(t) \geq 0, A_e(t) \geq$

3.5. Model Analysis

The nonlinear system of equations will be qualitatively analyzed so as to find the conditions for existence and stability of disease free equilibrium points. Analysis of the model allows us to determine the impact of public health education on the transmission of HIV/AIDS infection in a population.

3.5.1. Local Stability of Disease-Free Equilibrium (DFE)

The disease-free equilibrium of our model is

$$Y_0 = (S_u^*, I_u^*, A_u^*) = (-, 0, 0) \qquad (3)$$

It can be seen that Y_0 attracts the region (stable manifold of Y_0)

$$D_u = \{(S_u, I_u, A_u) \in \mathbb{R}_+^3 : Nu \leq -\} \qquad (4)$$

By using the next generation method, the matrices F and V for the new infection terms and the remaining transfer terms respectively, are given by

$$F = \begin{pmatrix} — & — \end{pmatrix}$$

and

$$V = \begin{pmatrix} u & \end{pmatrix}$$

It follows that the reproduction number, denoted by ,
is given by

$$(F \quad) = \frac{\beta[\mu \quad u]}{(\sigma \quad \eta)(\eta \quad u)} \qquad (5)$$

The *DFE*, Y_0, of the education-free model is locally asymptotically stable (*LAS*) if $\mathcal{R}_0 < 1$, and unstable if $\mathcal{R}_0 > 1$.

The quantity $\mathcal{R}_0$ is the reproduction number which measures the average number of new cases of HIV infection generated by a single HIV infected individual in a totally uneducated population.

3.5.2. Existence of Endemic Equilibrium

The education-free model has a unique positive endemic equilibrium point (EEP), where at least one of the infected components (I_u or A_u) is non-zero, given by $Y^{**} = (S_u^{**}, I_u^{**},$).

To establish this, the equations in model (2) are solved in terms of the force of infection at steady-state (λ_u^{**}), given by

$$= \frac{\beta(I_u^{**} + \eta_u A_u^{**})}{} \qquad (6)$$

Setting the right hand sides of the model (2) to zero gives

$$\overline{(\lambda \qquad \mu) + (\sigma \qquad)} \qquad (7)$$

$$\overline{(\delta + \mu)(\lambda_u^{**} + \mu)(\sigma_u + \mu)} \Big\}$$

The education-free model (2) has a unique positive endemic equilibrium wherever $\mathcal{R}_0 > 1$, and no positive endemic equilibrium whenever $\mathcal{R}_0 < 1$.

3.5.3. Global Stability of the Endemic Equilibrium

The education-free model (2) shows the existence of a unique endemic equilibrium if $\mathcal{R}_0 > 1$.Now study the global stability of the unique positive endemic equilibrium

$$= (S_u^{**}, I_u^{**}, A_u^{**})$$

When the disease-induced mortality rate is negligible, the endemic equilibrium of education-free model (2) with $\delta_u = 0$ is globally asymptotically stable (*GAS*) in D_u / Y_0 whenever $\mathcal{R}_0 > 1$.

Since $S_u = N_u - I_u - A_u$ and $N_u \to \Pi/\mu$ as $t \to \infty$, substituting these in last two equations of model (2) with $\delta_u = 0$ gives

$$\left[\frac{}{u} \qquad \right] - (\sigma \qquad u) \Big\} \qquad (8)$$

In conclusion, for $\mathcal{R}_0 > 1$, the disease remains endemic in education-free model (2) with negligible mortality rate induced by the disease.

3.5.4. Existence of Steady States of the system

The equilibrium points of the system can be obtained by equating the rate of changes to zero.

IV. SIMULATION AND DISCUSSION

This section presents results based on simulated data on epidemic models discussed in earlier with a view of developing algorithms which will later help in validating the real HIV/AIDS epidemiological data in Kenya. The Poisson distribution was taken for the birth and death rates corresponding to the life expectancy in Kenya of 67 years.

4.1. Empirical Results

In this section we present results when the epidemic model was subjected to the real HIV/AIDS data reported to the national HIV/AIDS program for the period 2000-2017. The results in this section relate to the application of the HIV/AIDS uneducated model.

4.2. Model Validation

To estimate the parameter values and validate the model, we fit the system to the data for the estimated new HIV infection in Kenya. A Matlab code is used where unknown parameter values are given a lower and upper bound from which the set of parameter values that produce the best fit are obtained. The assumption we took was that 25 million were uneducated susceptible individuals, 600,000 were uneducated infected individuals, 300,000 were uneducated full blown Aids individuals and 40,000 were those who had recovered from the AIDS symptoms. Hence, the model (1) can be used to gain realistic insight into HIV transmission dynamics in the presence of public health education campaign.

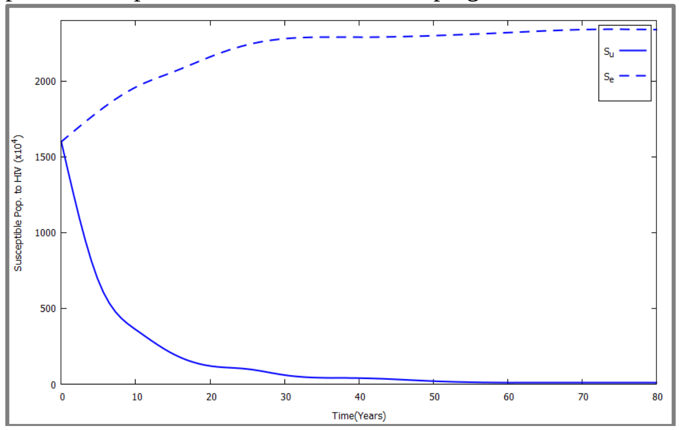


Figure 1: HIV/AIDS Model with and without Control representation with Observed Data for the Susceptible Number of Cases in Kenya

It shows the variations of the susceptible population with and without control. Without control this is when there is no knowledge on the best methods of protecting oneself from infection of the HIV disease the level of infection is very high and the susceptible population that was initially 16 million drops drastically up to the 20th year.

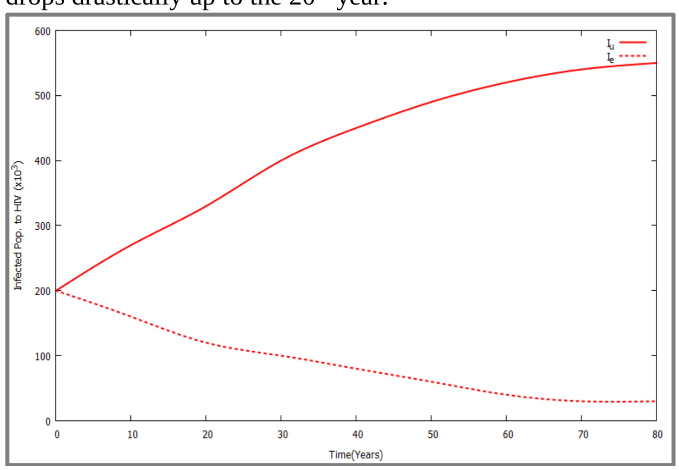


Figure 2: HIV/AIDS Model with and without Control representation with Observed Data for the Infected Number of Cases in Kenya

It shows the variations of the infected population with and without control. Without control this is when there is no knowledge on the best methods of managing oneself from developing symptoms after infection of the HIV disease.

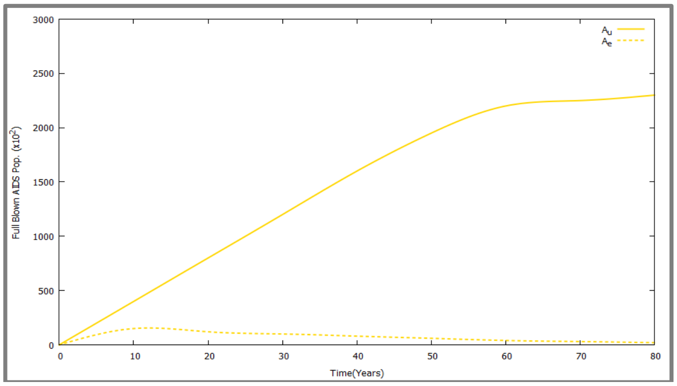


Figure 3: HIV/AIDS Model with and without Control representation with Observed Data for the Full Blown Aids Number of Cases in Kenya

It shows the variations of the full blown Aids population with and without control. Without control, those showing symptoms of infection increase drastically to a population of 0.025 million at the 54^{th} year due to lack of proper health education in disease management skills.

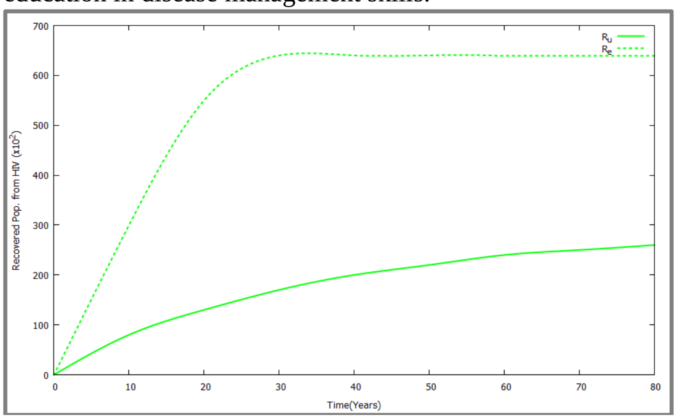


Figure 4: HIV/AIDS Model with and without Control representation with Observed Data for the Recovered Number of Cases in Kenya

It shows the variations of the recovered population from opportunistic diseases with and without control. Without control this is when there is no knowledge on the best methods of managing oneself from developing symptoms from opportunistic diseases after infection of the HIV disease.

V. Conclusion

The analytical results showed that our model is locally asymptotically stable when the basic reproduction number, < 1. The basic reproduction number, $R_{0,}$ can solve concrete problems since it completely determines the stability dynamics and outcome of the disease. It was found that if the threshold parameter is less than one the disease free equilibrium is stable and the disease dies out. However, if the threshold parameter is more than one, there exists a unique endemic equilibrium that is locally asymptotically stable. Theoretical determination of threshold conditions for R_0, is of important public heath interest. We reach total agreement with WHO and UNICEF recommendations on HIV/AIDS control, as herd resistance increases with number of opportunity. Any other limitation is the lack of success when prospecting global stability for SEIR epidemiological models with non-constant population. This is directed to the stakeholders, public health agencies health care providers and the various county governments to enable them determine how best to allocate scarce resources for HIV/AIDS prevention and management in the country. The model has shown success in attempting to predict the causes and reason for rapid spread of HIV/AIDS transmission within a certain population.

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