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Numerical Analysis of a Mathematical Model for Absorption of Cosmetic Drugs Formulations through the Skin

Mayaka Augustine Ayanga*, Johana K. Sigey**, Jeconiah A. Okelo***, James M. Okwoyo**** &
Kang'ethe Giterere*****

*Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, KENYA.
E-Mail: ayangaaugustine{at}gmail{dot}com
**Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, KENYA.
E-Mail: jksigey{at}jkuat{dot}ac{dot}ke
***Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, KENYA.
E-Mail: jokelo{at}jkuat{dot}ac{dot}ke
****School of Mathematics, University of Nairobi, Nairobi, KENYA. E-Mail: jmkwoyo{at}uonbi{dot}ac{dot}ke
*****Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, KENYA.
E-Mail: kgiterere{at}jkuat{dot}ac{dot}ke

***Abstract*—**The mathematical model for absorption of cosmetics drug formulations through human skin is studied. The study is concerned with the structure of the skin, since it has structures that enhance absorption of drugs through its permeable. Liposome cosmetic drug is used in this study. A partial Differential equations describing Fick's second law is solved using Finite Difference Method. The equation is used to determine mathematically absorption these cosmetic drug formulations through the skin. The Central Difference numerical scheme is developed for this work. Numerical results obtained for the absorption model through the skin are presented graphically to determine the effects of membrane thickness, membrane partition coefficient, membrane permeability coefficient and membrane diffusion coefficient on cosmetic drug absorption formulations through the human skin and their effect to the body. From the simulated results have been found that for the particular time, drug absorption decreases with increase in membrane thickness (L) and partition coefficient (K). Liposome Drug absorption through the skin increases with increases in permeability coefficient (p) and also diffusivity coefficient (d) at a particular time.

***Keywords*—**Crank Nicolson Numerical Scheme; The Partial Differential Equations; Finite Difference Method; Costimetic Drug; Transdermal Drug Delivery (TDD); Membrane Thickness; Partition Coefficient; Permeability Coefficient and Diffusivity Coefficient.

***Abbreviations*—**Forward Difference Scheme (FDS); Partial Differential Equation (PDE); Stratum Corneum (SC); Transdermal Drug Delivery (TDD).

I. INTRODUCTION

1.1. Introduction

A cosmetic is any substance applied to the skin to enhance the external colour. Good examples of cosmetics are; ointments, lotions, hair dyes, lipsticks and eye line. When skin is exposed to a drug (e.g. cream/ointment is applied or environmental exposure to toxins) partitioning into the stratum corneum and diffusion in it occur that results in flux through the stratum corneum which in turn increase concentration of drug in the viable epidermis this concentration crucial in determining skin absorption. The process of transport through the layers of skin (i.e. stratum corneum, viable epidermis and dermis) is time dependent at first and described by diffusion equation. Once absorbed, a cosmetic formulation is distributed throughout the body by means of the circulation of the blood. The distribution of most cosmetic formulations in the body is far from even. This complicates the efforts to correlate blood levels and the pharmacological effects of the cosmetic formulations used; some cosmetic formulations tend to bind to blood elements, some cosmetic formulations dissolve more readily in body fat depots and a few cosmetic formulations have a strong tendency to locate in bone. Bronaugh [1] indicated that cosmetic formulations must be very fat soluble to enter the brain. It is generally true that high blood cosmetic formulation levels yield correspondingly greater pharmacological effects. Even though a cosmetic formulation is in the bloodstream it must pass across various barriers to reach its site of action. Only a very small proportion of the total amount of cosmetic formulation in a body at any one time is in direct contact with the specific cells that produce the pharmacological effect. Most of the cosmetic formulation is to be found in areas of the body remote from the cosmetic formulation's site of action. In the case of psychoactive cosmetic formulations, most of the cosmetic formulation is to be found outside the brain and is therefore not directly contributing to the pharmacological effect. Cosmetic formulation that has accumulated in a given tissue may serve as a reservoir that prolongs cosmetic formulation action in that same tissue or at a distant site reached through circulation. Skin has been considered as a promising route for the administration of drugs because of its accessibility and large surface area. Transdermal drug delivery system, designed to deliver a variety of drugs to the body through diffusion across the skin layers, is appealing for several reasons including avoidance of the variable absorption and metabolic breakdown associated with oral treatments, drug administration can be continuous, and minimal intestinal irritation can be avoided.

1.2. Liposome

Liposome is commonly used in transdermal drug delivery system because of its much higher diffusivity in skin compared to most bare drugs. In addition, degradation of liposome is easily controlled, so it has been utilized in many skin products. Liposomes represent yet another "chemical" method frequently employed to enhance delivery into the skin, especially in the case of cosmetics and moisturizers. While intact liposomes probably do not penetrate the stratum corneum, they can be used to increase effective drug solubility in a vehicle (C *v)* and facilitate partitioning into the skin (K_m).

1.3. Geometry of the Problem

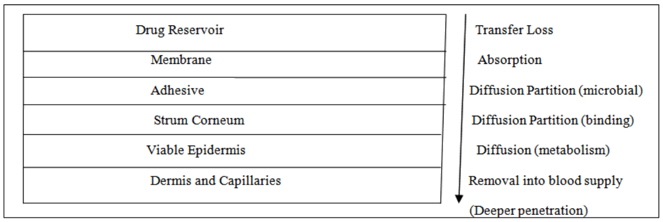

Figure 1: Absorption and Loss Processes of Drug into the Skin TDD System. (Modified from Hadgraft [6])

A basic knowledge of absorption process from the transdermal patches and into and through the skin is important, figure 1 represents the idealized scheme of these consecutive steps. As can be seen in figure 1, the predominant events involve partition and diffusion. The drug will partition from reservoir into polymer matrix that comprises the rate-limiting membrane. Once in the membrane, diffusion will occur down a concentration gradient at a rate which will be controlled by the diffusion coefficient of drug in the polymer. Drug has diffused through the rate controlling membrane, it will partition into the adhesive layer. Transport through this region will also be related to molecular size, but will be considerably faster than that through the membrane. Drug contained in this area will be released relatively rapidly to the skin. Typically, the release profile will follow first-order kinetics [Hadgraft, 6]. The exact amount of drug required is dependent on physicochemical and biological properties of the compound. If these are known, educated estimate will be made. Without a precise knowledge of absorption and degree of storability of drugs, we cannot make effective transdermal patches. For the knowledge of drug concentration into the human dermal part we established here a mathematical model. This model is depending on physicokinetic properties of drug [Hadgraft, 6].

1.4. Drug Absorption through the Skin

Any substance that come into contact with the skin surface can penetrate the stratum corneum, permeate the skin strata and diffuse into the underlying tissues or become absorbed into the dermal blood supply and hence the systemic circulation as shown in figure 1. Drug absorption through the skin can be manipulated in order to deliver drug or substance to:

i. the skin surface (e.g. cosmetics, sunscreens, insect repellents).
ii. Targeted skin layers (e.g. corticosteroids to treat localised skin disorders).
iii. Underlying tissues (e.g. non steroidal anti-inflammatory agents to treat muscular inflammation).
iv. the systemic circulation (e.g. oestradiol to treat symptoms of menopause, nicotine to aid smoking cessation, glyceryltrinitrate for the treatment of angina pectoris and fentanyl for the treatment of chronic cancer-related pain).

However, TDD also has limitations, which are mainly related to the biological, physicochemical and pharmacokinetic aspects of percutaneous absorption. For instance, due to the efficient barrier properties of the stratum corneum there will be a characteristic lag time prior to the accumulation of adequate plasma drug concentrations and the onset of therapeutic action. Thus, TDD is not suitable for acute conditions where a rapid therapeutic effect is desired. The partitioning of a drug into the SC (stratum corneum) and its subsequent diffusion through this integument are critical determinants of percutaneous drug absorption.SC and epidermis are quite distinct structures in that the SC essentially provides a lipophilic milieu for drug diffusion whereas the epidermis provides a more hydrophilic domain. As the epidermis does not contain a local blood supply, it essentially serves as an unstirred aqueous boundary layer for any drug with high lipid solubility and low aqueous solubility. Epidermis also contains a variety of enzymes; a sufficient amount of the drug may not reach the underlying skin strata if the drug is susceptible to enzymatic degradation. Partitioning into the hypodermis and underlying muscles can occur if the drug bypasses the dermal blood supply. As the hypodermis is predominantly composed of adipose tissue, it is possible that lipophilic drugs may accumulate within this layer if they are not cleared by the dermal blood supply [Davis, 3].

1.5. Related Works

The skin is a complex organ that serves to protect humans from chemical, physical, and biological intrusion, while retaining moisture and providing thermal regulation. It consists of three primary regions: the epidermis, the dermis, and the hypodermis. The epidermis is the outermost layer of the skin in contact with the environment, ranging between 0.075 and 0.20 mm thick in most regions and between 0.4 and 0.6 mm thick in the palms and soles. Gujral et al., [5] studied on the human skin to determine on what entails and how chemicals can flow through the skin. The study established that a skin is like a tightly woven fabric, seemingly impervious but porous at the microscopic level. Through its millions of tiny openings, the body oozes sweat and absorbs some substances applied to the skin. Now, researchers are developing techniques to move a wider range of cosmetic formulations across the skin barrier. Some work by making the skin more permeable, either by widening its pores through ultrasound waves or by softly puncturing the skin with a grid of microscopic needles. Other techniques use a mild electric current to propel the cosmetic formulation through the skin or tag cosmetic formulation molecules with compounds that help it slip in through pores. Cosmetic formulation delivery with these techniques offers several advantages over pills and injections. It can ensure the steady release of medication into the patient's bloodstream over long periods, improving the efficacy of a dose. It can prevent the rapid breakdown that many cosmetic formulations taken orally undergo when they pass through the digestive system. Dressler [4] studied on what the skin is comprised of. His study established that there are three primary regions, the outer epidermis and middle dermis and the lowest area, the hypodermis. The epidermis consists of multiple strata (i.e. layers,) with its superficial the most crucial in prevention of skin penetration. This outermost layer of the epidermis, the stratum corneum (flattened cells, also known as the horny layer,) serves as your body’s primary defense. The stratum corneum, thick with dead skin cells (mostly keratin) and various waxy substances, acts as a wall of protection from external moisture, chemicals, UV radiation and is your foremost guard against the bacterial world. As these dead bits fall of, or exfoliated from the body, the lower layers of skin replenish the surface with additional keratin.

Scharf et al., [15] indicated that skin absorption is a route by which substances can enter the body through the skin. Along with inhalation, ingestion and injection, dermal absorption is a route of exposure for toxic substances and route of administration for medication. Absorption of substances through the skin depends on a number of factors, the most important of which are concentration, duration of contact, solubility of medication, and physical condition of the skin and part of the body exposed. Skin (percutaneous, dermal) absorption is a term that describes the transport of chemicals from the outer surface of the skin both into the skin and into the systemic circulation. Until the beginning of the 20th century, the skin was thought to be completely inert and impermeable to chemicals that might otherwise enter the body, however we now know many chemicals can get through the skin. Skin absorption relates to the degree of exposure to and possible effect of a substance which may enter the body through the skin. Human skin comes into contact with many agents intentionally and unintentionally. Scharf et al., [15] in his paper absorption can occur from occupational, environmental or consumer skin exposure to chemicals, cosmetics, or pharmaceutical products. Some chemicals can be absorbed in enough quantity to cause detrimental systemic effects. Skin disease (dermatitis) is considered one of the most common occupational diseases. In order to assess if a chemical can be a risk of either causing dermatitis or other more systemeic effects and how that risk may be reduced one must know the extent to which it is absorbed, thus dermal exposure is a key aspect of human health risk assessment.

Ruocco & Wolf (2001) on their paper looked on how inhalation, ingestion and injection, dermal absorption is a route of exposure for bioactive substances including medications. Absorption of substances through the skin depends on a number of factors: Concentration, Duration of contact, Solubility of medication, Physical condition of the skin and part of the body exposed including the amount of hair on the skin. Morganti et al., [8] looked at how the skin Penetration represents the amount of a topically chemical that exists between the top layer (stratum corneum) and the bottom layer (stratum basale). During penetration, the body does not yet absorb the chemical, and it cannot affect the body systems. Skin Absorption occurs when the topically applied chemical breaks the skin barrier to reach the bloodstream. Whether this chemical becomes a risk is determined by what occurs after absorption. You body can filter out the chemical via bodily fluids, or bioaccumulation (build up) occurs. Many variables affect the speed (or probability) of penetration and absorption. First, the composition of the chemical to which skin is exposed. The area of skin that is exposed (thinner-skinned areas are more susceptible to penetration and thicker skin is less) and the condition of the skin are all significant factors.

Rauma et al., [10] looked at the composition of the chemical exposed to skin determines its possibility of entering the skin primarily the molecule size and solubility of the chemical. He looked how the design of cosmetic and skin care formulas is to benefit the outer layer of skin absorption into the body would waste the effects of these products. Antioxidants in skin care won’t do their job if they don’t stay in the layers of skin it is challenging enough to develop a formula that enables an ingredient to penetrate the surface layer. The majority of cosmetics are not soluble in skin (i.e. lipid, or fat-soluble) and are too large in molecule to fit through the stratum corneum. Precisely because of these qualities, some skin care formulas require specially developed “penetration enhancers” to deliver ingredients like vitamin C or retinol. These types of medicine require formulation specifically for this purpose, requiring chemical engineering to create a molecule that is soluble in skin, and small enough to penetrate and absorb into the body. Much controversy has arisen over the ingredients in skin care products that inadvertently absorb into our body and the possible risk to our health. Rauma et al., [10] on their paper looked and determined the safety of a chemical in skin absorption is about risk assessment. The toxicity of an ingredient is in the amount absorbed and accumulated, or “the dose makes the poison.”The nutrients and substances depend on our health can kill us if absorbed large enough amount. When a chemical penetrates through the skin and is absorbed into human bodies, which may be converted into another chemical form, metabolized or accumulate. At the dose in which a chemical becomes harmful (toxic) is the threshold, less than this amount is safe, and more becomes a dangerous. Human body is designed to break down chemicals into other forms that are easily excreted via fluids. The threshold is the over/under amount of human body’s ability to process a chemical and still keep the body healthy.

Morganti et al., [8] on their paper on the factors that affect a cosmetic formulation’s distribution throughout an organism, but they considered that the most important ones are the following: an organism’s physical volume, the removal rate and the degree to which a cosmetic formulation binds with plasma proteins and / or tissues. This concept is related to multi-compartmentalization. Any cosmetic formulations within an organism will act as a solute and the organism’s tissues will act as solvents. The differing specificities of different tissues will give rise to different concentrations of the cosmetic formulation within each group. Therefore, the chemical characteristics of a cosmetic formulation will determine its distribution within an organism. For example, a liposoluble cosmetic formulation will tend to accumulate in body fat and skin-soluble cosmetic formulations will tend to accumulate in extracellular fluids. The volume of distribution (V_d) of a cosmetic formulation is a property that quantifies the extent of its distribution. It can be defined as the theoretical volume that a cosmetic formulation would have to occupy (if it were uniformly distributed), to provide the same concentration as it currently is in blood plasma. It can be determined from the formula:$V_d = \frac{A_b}{C_p}$; where: $A_b$is total amount of the cosmetic formulation in the body and C_p is the cosmetic formulation’s plasma concentration. As the value for A_b is equivalent to the dose of the cosmetic formulation that has been administered the formula shows us that there is an inversely proportional relationship between V_d and C_p. That is, that the greater C_p is the lower V_d will be and vice versa. It therefore follows that the factors that increase C_p will decrease V_d. This gives an indication of the importance of knowledge relating to the cosmetic formulation’s plasma concentration and the factors that modify it.

Vineeta et al., (2012) presented model that discusses the drug distribution in the layers of human dermal parts. They considered tissue absorption rate of drug is decreasing function of drug concentration, also considered linear shape functions for approximating element wise approximate solution by Finite Element Method (FEM).In this method each region was referred to as an element and the process of subdividing a domain into a finite number of elements is referred to as discretization. Elements are connected at specific points, called nodes, where the solution was taken to be continuous along common boundaries of adjacent elements. Khanday & Aasma [7] attempted to establish a mathematical model for the diffusion of drugs through the transdermal drug delivery system. The model identifies the pattern of drug diffusion in human body and its effective absorption rates at various compartments of skin and sub-cutaneous tissues. The finite element method was used to obtain the solution of the mass diffusion equation with appropriate boundary conditions. The tissue absorption rate of drug has been taken as the decreasing function of drug concentration from the skin surface towards the target site. The concentration at nodal points was been calculated which in turn determines the drug absorption at various layers. It was observed that due to dense network of connective tissues in dermal and sub-dermal parts, the drug absorption was maximum as compared to cutaneous tissues.

1.6. *Statement of the Problem*

Various mathematical models and methods have been used to investigate absorption of drugs through the skin. These models that use Finite Element Method include Vineeta et al., (2012), Khanday & Aasma [7]. The Absorption of Liposome drug through the human skin using Fick’s model Equation by Finite Difference Method has not yet been investigated. There is need to study how these parameters affect absorption of drugs and their products into the skin in order to determine and investigated their effects to the human body. Finite Difference Method is used to investigate how the membrane thickness, membrane partition coefficient, membrane permeability coefficient and membrane diffusion coefficient affect concentration and absorption of the Liposome drug through the skin.

1.7. *Specific Objectives of the Study*

The specific objectives of the study are to determine the;

i) effects of membrane thickness on Liposome drug absorption through the skin
ii) effects of membrane partition coefficient on Liposome drug absorption through skin
iii) effects of membrane permeability coefficient on Liposome drug absorption the skin
iv) effects of membrane diffusion coefficient on Liposome drug absorption

1.8. *Justification of the Study*

The use of Fick’s model for Liposome drug absorption can provide valuable information concerning the mechanisms of action of different drug formulation components and their contribution to absorption through the skin. Once the concentration of Liposome cosmetic formulations is determined through skin absorption; the concerned government agencies will be notified of the same so that the parties involved in human health can take appropriate measures to enhance hygiene through health education to citizens. The general public using and handling cosmetic these formulations will be alerted to be aware of the dangers encountered when using and handling cosmetic formulations. Proper measures will be put in place to regulate disposal of used cosmetics in order to minimize its absorption through human skin.

The resulting investigations on these parameters will act as an opening call to all stakeholders, including nutritionist, government agencies both at county and national level.

II. MATHEMATICAL FORMULATION

2.1. Chapter Overview

In this chapter, the equations governing Formulation of drug absorption and diffusion mathematical model is be discussed.

2.2. Parameters Controlling Absorption

Conventional transdermal drug delivery is a passive process governed by Fick's law, that is, the rate of absorption or flux (*J*) of any substance across a barrier is proportional to its concentration difference across that barrier; Michael (1975) and Franz (1983). For topically applied drugs, the concentration difference can be simplified as the concentration of drug in the vehicle, C_v, and the proportionality constant relating flux to concentration is the permeability coefficient, K_p (equation 1). K_p is composed of factors that relate to both drug and barrier, as well as their interaction. These factors are, K_m the partition coefficient; *D*, the diffusion coefficient; and *L*, the length of the diffusion pathway (equation 2). Thus, four factors control the kinetics of percutaneous drug absorption (equation 4); however, it is of great practical importance that two of the four (C_v,K_m) are highly dependent on one additional factor, the vehicle.

$$J = k_P C_v \tag{1}$$

$$J = \left(\frac{D K_m}{L} \right) \tag{2}$$

2.3. Formulation of Periodic Drug Distribution Mathematical Model

In 1855, Fick recognized that a mathematical equation of heat conduction developed by Fourier in 1822 could be applied to mass transfer. Fick found out that the flux J is proportional to

Concentration gradient $\frac{\partial C}{\partial x}$, hence

$$J = -D \frac{\partial C}{\partial x} \tag{3}$$

Where D is diffusivity, C is concentration and x is distance of movement perpendicular to the surface of the barrier. Equation (3) is known as Fick's first law. Fick's second law, Patrick J. Sinko [9] was derived, as the concentration in a particular volume element changes only as a result of net flow of diffusing molecules into or out of the region i.e

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \tag{4}$$

This equation represents diffusion only in x direction. Extended form of Equation (4) was employed by Chao et al., [2] and Chao and Yang (1975) to study heat diffusion in dermal region. The diffusion of chemical compounds in the membrane is expressed theoretically by Fick's 2nd law of diffusion in equation (4) under the assumption that the membrane is a homogeneous single layer. When a sink condition is assumed on the receiver side of the membrane, *i.e.*, $x = L$, a set of initial conditions

$$C = 0 \text{ at } t = 0 \text{ ; } 0 < x < L \tag{5}$$

and boundary conditions

$$C = KC_v \text{ at } x = 0 \text{ and } C = 0 \text{ at } x = L, \tag{6}$$

Where *K* is the partition coefficient of the penetrant from the vehicle to membrane and C_v is the penetrant concentration in the vehicle) are obtained. Similar equation has been widely used by Saxena and his coworkers [11-14]. In most of the cases Finite element method has been employed by Saxena [17]. Saxena & Sharma [11] used this approach to solve One-dimensional drug distribution problem in human dermal region. We use this approach in present model and consider here when drug is diffused into dermal layers, drug mainly absorbed by tissues and blood but tissue absorption rate of drug will be decreasing function of drug concentration and drug absorption by other factors almost negligible.

2.4. Determination of Membrane Permeation

Since the membrane is supposed to be homogenous in one layer, the permeation profiles of drugs throughout the membrane are analyzed using a one-layered diffusion model; Scheuplein et al., [16] and Watanabe et al., (2001). Under the initial and boundary conditions shown as above in equation (5) and (6), the amount of drug permeating the unit area of the skin membrane at time *t*, *Q*, can be represented as,

$$Q = KLC_V \left\{ \frac{D}{L^2} t - \frac{1}{6} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \exp\left(-\frac{Dn^2\pi^2}{L^2} t \right) \right\} \tag{7}$$

The partition parameter (*K*) and diffusion parameter *(D)* are obtained by curve fitting the obtained data to Equation (5) using the least squares method. In the calculation of *D* and *K*, the thickness of the skin membrane are fixed at 48 μm, 68 μm and 1088 μm. Permeability coefficient, *P*, was calculated by an Equation,

$$P = \frac{KD}{L} \tag{8}$$

III. METHOD OF SOLUTION

3.1. Introduction

In this section, the method and procedure of solving the problem are discussed. A system of partial differential equations with appropriate boundary conditions governing our problem is solved using a Finite Difference Method. The diffusion of chemical compounds in the membrane is expressed theoretically by Fick's 2^{nd} law of diffusion under the assumption that the membrane is a homogeneous single layer, where *C* is the penetrant absorption in the membrane at position *x*, and time, *t*.

3.2. Computational Procedure

In this study Central Difference numerical scheme is developed and Finite Difference Method is used solve the Fick's 2^{nd} law of diffusion. The method obtains a finite system of linear or nonlinear algebraic equations from the Fick's Partial Differential Equation by discretizing the given equation and coming up with the numerical schemes analogues to the equation, in our case the Fick's Law of diffusion. We solve the equations subject to the given boundary conditions. MATLAB software is used to generate solution values in this study.

3.3. Effect of Skin Membrane Thickness

Considering Fick's 2^{nd} law of diffusion in equation (2), we substitute for$D=\frac{PL}{K}$, from equation (8) where P is permeability coefficient, D is diffusion coefficient, K is the partition coefficient of the drug penetrant and L is the skin membrane thickness. Using Central Difference numerical scheme, $C_t$is replaced bycentral difference approximation while C_t is replaced bycentral difference approximation, the equation (6)becomes

$$\frac{C_{i+1,j}-C_{i,j-1}}{2(\Delta x)}=\frac{PL}{2K}\left[\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}}{(\Delta x)^2}\right] \qquad (9)$$

We investigate the effect of P, L and K on the concentration of the Liposome drug into the skin. Taking$\Delta x=\Delta t=0.01$, K=1.2, P=0.02, and L=48 $^{\mu m}$ we get the scheme

$$0.131C_{i+1,j}-0.272C_{i,j}-0.136C_{i-1,j+1}=0.005C_{i,j-1} \qquad (10)$$

Taking and i=1,2,3…….8 and j=1we form the following systems of linear algebraic equations

$$0.131C_{2,1}-0.272C_{1,1}-0.136C_{0,1}=0.005C_{1,0}$$
$$0.131C_{3,1}-0.272C_{2,1}-0.136C_{1,1}=0.005C_{2,0}$$
$$0.131C_{4,1}-0.272C_{3,1}-0.136C_{2,1}=0.005C_{3,0}$$
$$0.131C_{5,1}-0.272C_{4,1}-0.136C_{3,1}=0.005C_{4,0}$$
$$0.131C_{6,1}-0.272C_{5,1}-0.136C_{4,1}=0.005C_{5,0}$$
$$0.131C_{7,1}-0.272C_{6,1}-0.136C_{5,1}=0.005C_{6,0}$$
$$0.131C_{8,1}-0.272C_{7,1}-0.136C_{6,1}=0.005C_{7,0}$$
$$0.131C_{9,1}-0.272C_{8,1}-0.136C_{7,1}=0.005C_{8,0}$$
$$0.131C_{10,1}-0.272C_{9,1}-0.136C_{8,1}=0.005C_{9,0}$$
$$0.131C_{11,1}-0.272C_{10,1}-0.136C_{9,1}=0.005C_{10,0}$$

The above algebraic equations can be written in matrix form as when C (0, t) =kC$_v$=0.0024 and C(x, 0) = 0

$$\begin{bmatrix} 0.272 & -0.131 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.136 & 0.272 & -0.131 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.136 & 0.272 & -0.131 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.136 & 0.272 & -0.131 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.136 & 0.272 & -0.131 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.136 & 0.272 & -0.131 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.136 & 0.272 & -0.131 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.136 & 0.272 & -0.131 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.136 & 0.272 & -0.131 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.136 & 0.272 \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \\ C_{51} \\ C_{61} \\ C_{71} \\ C_{81} \\ C_{91} \\ C_{101} \end{bmatrix} = \begin{bmatrix} 2.4\times10^{-3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$

Solving the above matrix equation using MATLAB, we get the solutions for changing L

L=68μm	L=88μm	L=108μm
$C_{1,1}=1.466009\times10^{-3}$	$C_{1,1}=1.993772\times10^{-3}$	$C_{1,1}=2.14399\times10^{-3}$
$C_{2,1}=1.211865\times10^{-3}$	$C_{2,1}=1.648136\times10^{-3}$	$C_{2,1}=1.893358\times10^{-3}$
$C_{3,1}=9.942758\times10^{-4}$	$C_{3,1}=1.352215\times10^{-3}$	$C_{3,1}=1.647407\times10^{-3}$
$C_{4,1}=8.06311\times10^{-4}$	$C_{4,1}=1.09661\times10^{-3}$	$C_{4,1}=1.405449\times10^{-3}$
$C_{5,1}=6.419889\times10^{-4}$	$C_{5,1}=8.731049\times10^{-4}$	$C_{5,1}=1.166805\times10^{-3}$
$C_{6,1}=4.958775\times10^{-4}$	$C_{6,1}=6.743934\times10^{-4}$	$C_{6,1}=9.308057\times10^{-4}$
$C_{7,1}=3.63116\times10^{-4}$	$C_{7,1}=4.938378\times10^{-4}$	$C_{7,1}=4\ 6.967859\times10^{-4}$
$C_{8,1}=2.391466\times10^{-4}$	$C_{8,1}=3.25239\times10^{-4}$	$C_{8,1}=4.60844\times10^{-4}$
$C_{9,1}=1.195733\times10^{-4}$	$C_{9,1}=1.626197\times10^{-4}$	$C_{9,1}=5.411771\times10^{-4}$
$C_{10,1}=0$	$C_{10,1}=0$	$C_{10,1}=0$

3.4. Effect of Permeability Coefficient on Drug Absorption

We investigate the effect of K on the absorption of the Liposome drug into the skin. Taking$\Delta x=\Delta t=0.01$, K=1.2, P=0.02, 0.04, 0.06 and D =0.2, we get the scheme

The above algebraic equations can be written in matrix form as when C (0, t) =kC$_v$=0.0024 and C(x, 0) = 0

$$\begin{bmatrix} 0.92 & -0.955 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.96 & 0.92 & -0.955 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.96 & 0.92 & -0.955 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.96 & 0.92 & -0.955 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.96 & 0.92 & -0.955 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.96 & 0.92 & -0.955 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.96 & 0.92 & -0.955 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.96 & 0.92 & -0.955 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.96 & 0.92 & -0.955 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.96 & 0.92 \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \\ C_{51} \\ C_{61} \\ C_{71} \\ C_{81} \\ C_{91} \\ C_{101} \end{bmatrix} = \begin{bmatrix} 2.4\times10^{-3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (12)$$

Solving the above matrix equation, we get the solutions for changing L

P=0.02	P=0.04	P=0.06
$C_{1,1}=2.105703\times10^{-3}$	$C_{1,1}=2.083729\times10^{-3}$	$C_{1,1}=2.043748\times10^{-3}$
$C_{2,1}=1.835303\times10^{-3}$	$C_{2,1}=1.798685\times10^{-3}$	$C_{2,1}=1.732616\times10^{-3}$
$C_{3,1}=1.601502\times10^{-3}$	$C_{3,1}=1.540384\times10^{-3}$	$C_{3,1}=1.459492\times10^{-3}$
$C_{4,1}=1.585548\times10^{-3}$	$C_{4,1}=1.304734\times10^{-3}$	$C_{4,1}=1.218089\times10^{-3}$
$C_{5,1}=1.353416\times10^{-3}$	$C_{5,1}=1.087973\times10^{-3}$	$C_{5,1}=1.0028\times10^{-3}$
$C_{6,1}=1.136076\times10^{-3}$	$C_{6,1}=8.86604\times10^{-4}$	$C_{6,1}=8.085677\times10^{-4}$
$C_{7,1}=7.35211\times10^{-4}$	$C_{7,1}=6.973418\times10^{-4}$	$C_{7,1}=6.307616\times10^{-4}$
$C_{8,1}=5.466832\times10^{-4}$	$C_{8,1}=5.170563\times10^{-4}$	$C_{8,1}=4.650666\times10^{-4}$
$C_{9,1}=3.628805\times10^{-4}$	$C_{9,1}=3.427209\times10^{-4}$	$C_{9,1}=3.073762\times10^{-4}$
$C_{10,1}=1.814402\times10^{-4}$	$C_{10,1}=1.713604\times10^{-4}$	$C_{10,1}=1.536881\times10^{-4}$

3.5. Effect of Skin Membrane Diffusivity Coefficient

We investigate the effect of D on the absorption of the Liposome drug into the skin. Taking$\Delta x = \Delta t = 0.01$, K=1.2, P=0.02, and D =0.2, 0.3, 0.4 we get the scheme

The above algebraic equations can be written in matrix form as when C (0, t) =kC$_v$=0.0024 and C(x, 0) = 0

$$\begin{bmatrix} 0.4 & -0.195 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.2 & 0.4 & -0.195 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0.4 & -0.195 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.2 & 0.4 & -0.195 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & 0.4 & -0.195 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0.4 & -0.195 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 & 0.4 & -0.195 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0.4 & -0.195 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0.4 & -0.195 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0.4 \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \\ C_{51} \\ C_{61} \\ C_{71} \\ C_{81} \\ C_{91} \\ C_{101} \end{bmatrix} = \begin{bmatrix} 2.4 \times 10^{-3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

Solving the above matrix equation, we get the solutions for changing L

D=0.2	D=0.3	D=0.4
$C_{1,1} = 2.048295 \times 10^{-3}$	$C_{1,1} = 2.087148 \times 10^{-3}$	$C_{1,1} = 2.10844 \times 10^{-3}$
$C_{2,1} = 1.740092 \times 10^{-3}$	$C_{2,1} = 1.804369 \times 10^{-3}$	$C_{2,1} = 1.839879 \times 10^{-3}$
$C_{3,1} = 1.468604 \times 10^{-3}$	$C_{3,1} = 1.54738 \times 10^{-3}$	$C_{3,1} = 1.591207 \times 10^{-3}$
$C_{4,1} = 1.227812 \times 10^{-3}$	$C_{4,1} = 1.312261 \times 10^{-3}$	$C_{4,1} = 1.35953 \times 10^{-3}$
$C_{5,1} = 1.012327 \times 10^{-3}$	$C_{5,1} = 1.0954 \times 10^{-3}$	$C_{5,1} = 1.142129 \times 10^{-3}$
$C_{6,1} = 8.172747 \times 10^{-4}$	$C_{6,1} = 8.934285 \times 10^{-4}$	$C_{6,1} = 9.364342 \times 10^{-4}$
$C_{7,1} = 6.381767 \times 10^{-4}$	$C_{7,1} = 7.031768 \times 10^{-4}$	$C_{7,1} = 7.39988 \times 10^{-4}$
$C_{8,1} = 4.708499 \times 10^{-4}$	$C_{8,1} = 5.216189 \times 10^{-4}$	$C_{8,1} = 5.5504238 \times 10^{-4}$
$C_{9,1} = 3.113057 \times 10^{-4}$	$C_{9,1} = 3.458247 \times 10^{-4}$	$C_{9,1} = 3.654266 \times 10^{-4}$
$C_{10,1} = 1.556528 \times 10^{-4}$	$C_{10,1} = 1.729123 \times 10^{-4}$	$C_{10,1} = 1.827133 \times 10^{-4}$

IV. RESULTS AND DISCUSSION

4.1. Chapter Overview

The simulated results shows relationships between Liposome drug concentration and various parameters as obtained by numerical computation are given in figures 2, 3, 4 and 5. The simulation results given focus on the effects of the diffusion coefficient D, the partition coefficient K, tissue thickness L and permeability coefficient P.

4.2. Effects of Membrane Thickness on Drug Absorption

We solve equation (10) using MATLAB and get the results of the effects of membrane thickness on drug concentration as shown in table 1 one below

Table 1: Values Drug Absorption for Varying Membrane Thickness

Distance (x)	L=68 μm	L=88 μm	L=108 μm
0	1.466009*10^-3	1.993772*10^-3	2.14399*10^-3
1	1.211865*10^-3	1.648136*10^-3	1.893358*10^-3
2	9.942758*10^-4	1.352215*10^-3	1.647407*10^-3
3	8.06311*10^-4	1.09661*10^-3	1.405449*10^-3
4	6.419889*10^-4	8.731049*10^-4	1.166805*10^-3
5	4.958775*10^-4	6.743934*10^-4	9.308057*10^-4
6	3.63116*10^-4	4.938378*10^-4	6.967859*10^-4
7	2.391466*10^-4	3.25239*10^-4	4.60844*10^-4
8	1.195733*10^-4	1.626197*10^-4	2.320422*10^-4
9	0	0	0

The results in the table 1 above is represented graphically as seen in figure 2 below

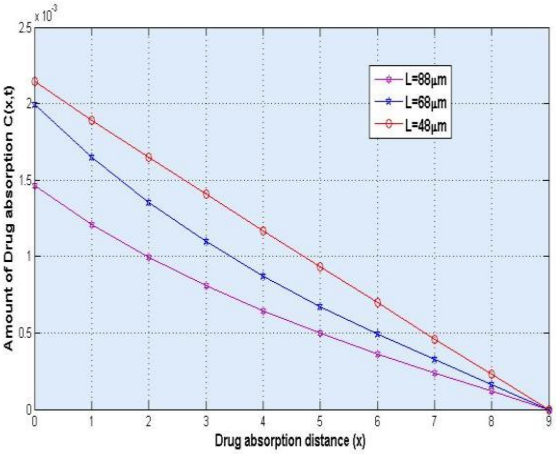

Figure 2: Graph of Drug Absorption against Drug Movement Distance at Varying Membrane Thickness

The effect of the tissue thickness on the overall mass transport of a Liposome drug across metabolizing tissue is depicted in figure 2.The steepness of curve in these figures decreases as we move from outer surface to inner boundary, which is due to the effect of inner boundary condition where absorption decreases with the increase in distance from the outer boundary. For very thin tissue sheets (i.e. 48 µm) the flux of substrate into the sheets is sufficiently high to replace metabolized fresh substrate almost completely. On the contrary, in thick cell sheets of 108 µm thickness, a typical thickness for the viable part of the human epidermis, a steep drop of the absorption gradient is observed. Obviously in thick tissue diffusion is not fast enough to fully replace metabolized substrate and thus becomes rate limiting.

4.3. Effects of Permeability Coefficient on Drug Absorption

We solve equation (10) using MATLAB and get the results of the effects of permeability coefficient on drug concentration as shown in table 1 one below

Table 2: Values Drug Absorption for Varying Permeability Coefficient

Distance (x)	P=0.08	P=0.06	P=0.04
0	2.27108*10^-3	2.083729*10^-3	2.043748*10^-3
1	2.11825*10^-3	1.798685*10^-3	1.732616*10^-3
2	1.94313*10^-3	1.540384*10^-3	1.459492*10^-3
3	1.747553*10^-3	1.304734*10^-3	1.218089*10^-3
4	1.53358*10^-3	1.087973*10^-3	1.0028*10^-3
5	1.303464*10^-3	8.86604*10^-4	8.085677*10^-4
6	1.059628*10^-4	6.973418*10^-4	6.307616*10^-4
7	8.046374*10^-4	5.170563*10^-4	4.650666*10^-4
8	5.411771*10^-4	3.427209*10^-4	3.073762*10^-4
9	2.720203*10^-4	1.713604*10^-4	1.536881*10^-4

The results in the table 2 above is represented graphically as seen in figure 3 below

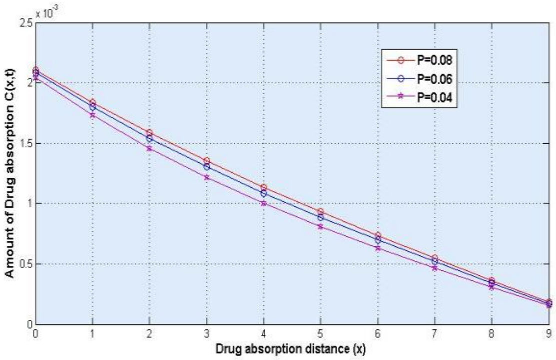

Figure 3: Graph of Drug Absorption against Drug Movement Distance at Varying Diffusion Coefficient

From figure 3 above, it is shown that as the skin permeability coefficient increases, the drug absorption also increases. For a particular value of permeability coefficient, the drug absorption also decreases through the skin penetration depth.

4.4. *Effects of Diffusion Coefficient on Drug Absorption*

We solve equation (10) using MATLAB and get the results of the effects of membrane thickness on Liposome drug concentration as shown in table 3 one below

Table 3: Values Drug Absorption for Varying Diffusion Coefficient

Distance (x)	**D=2** μmm^2 / sec	**D=4** μmm^2 / sec	**D=6** μmm^2 / sec
0	2.048295*10^-3	2.087148*10^-3	2.10844*10^-3
1	1.740092*10^-3	1.804369*10^-3	1.839879*10^-3
2	1.468604*10^-3	1.54738*10^-3	1.591207*10^-3
3	1.227812*10^-3	1.312261*10^-3	1.35953*10^-3
4	1.012327*10^-3	1.0954*10^-3	1.142129*10^-3
5	8.172747*10^-4	8.934285*10^-4	9.364342*10^-4
6	6.381767*10^-4	7.031768*10^-4	7.39988*10^-4
7	4.708499*10^-4	5.216189*10^-4	5.550424*10^-4
8	3.113057*10^-4	3.458247*10^-4	3.654266*10^-4
9	1.556528*10^-4	1.729123*10^-4	1.827133*10^-4

The results in the table 3 above is represented graphically as seen in figure 4 below

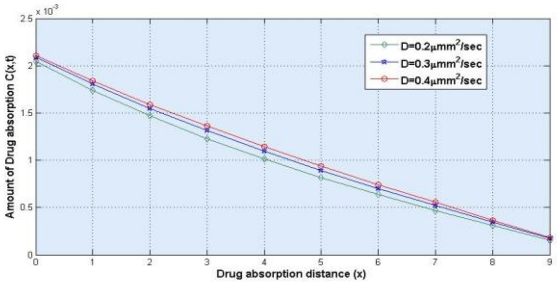

Figure 4: Graph of Drug Absorption against Distance at Varying Diffusion Coefficient

The effect of diffusivity coefficient can be observed from figure 4. Increase in diffusivity coefficient increases the Liposome drug absorption rate. The slope of this curve decreases at the interfaces; this effect is due to the change in properties of each sub-region of the skin.

4.5. *Effects of Membrane Partition Coefficient on Drug Concentration*

The results of the effects of membrane partition coefficient on drug concentration are shown in table 4 one below.

Table 4: Values Drug Concentration for Varying Membrane Partition Coefficient

Distance (x)	**K=1.2**	**K=1.4**	**K=1.6**
0	2.27108*10^-3	2.164229*10^-3	2.036014*10^-3
1	2.11825*10^-3	1.93355*10^-3	1.718687*10^-3
2	1.94313*10^-3	1.707421*10^-3	1.440746*10^-3
3	1.747553*10^-3	1.485309*10^-3	1.195822*10^-3
4	1.53358*10^-3	1.266692*10^-4	9.783029*10^-3
5	1.303464*10^-3	1.051055*10^-4	7.83203*10^-3
6	1.059628*10^-4	8.378918*10^-4	6.060515*10^-4
7	8.046374*10^-4	6.266999*10^-4	4.427886*10^-4
8	5.411771*10^-4	4.169825*10^-4	2.89673*10^-4
9	2.720203*10^-4	2.082463*10^-4	1.431957*10^-4

The results in the table 4 above is represented graphically as seen in figure 3 below

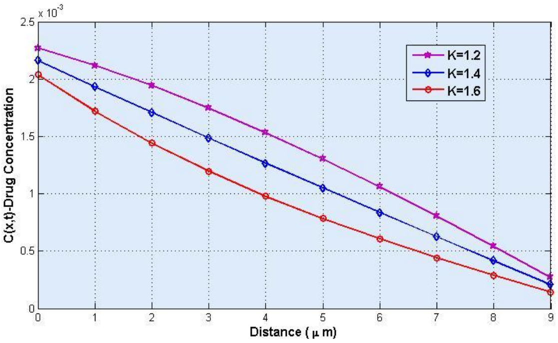

Figure 5: Graph of Drug Concentration against Distance at Varying Membrane Partition Coefficient

Figure 5 presents drug absorption gradients within the tissue for various values of the tissue partition coefficient K. At high drug concentrations within the tissue, membrane partition decreases. Thus, with increasing values of K, i.e K=1.6 the concentration gradient approaches a straight line and the tissue behaves like a passive membrane. For small values of K i.e K= 1.2,a lesser fraction of absorbed drug is metabolized and the gradients drop steeper. Hence, for efficient transdermal delivery the drug needs to partition well into the tissue or be administered at high concentration. This depends on the compound itself and on the vehicle selected for transdermal delivery. In general, topically applied drugs are poorly absorbed because only a small fraction partitions into the stratum corneum. Most of the drug remains on the skin surface, subject to loss from a multitude of factors (exfoliation, sweating, wash-off, rub-off, adsorption onto clothing, and chemical or photochemical degradation).In general, topically applied drugs are poorly absorbed because only a small fraction partitions into the stratum corneum. Most of the drug remains on the skin surface, subject to loss from a multitude of factors (exfoliation, sweating, wash-off, rub-off, adsorption onto clothing, and Chemical or photochemical degradation.

V. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

Fick's second Law was used numerically to model the simplified absorption of Liposome cosmetic drug across skin when an infinitely thin patch is placed on the outside of the stratum corneum. The homogeneous equation and boundary conditions that described the simplified model enabled solving the absorption problem using Finite Difference Method. From the simulated results, it has been found that for the particular time, drug absorption decreases with increase in membrane thickness (L) and partition coefficient (K). Liposome Drug absorption through the skin increases with increases in permeability coefficient (p) and also diffusivity coefficient (D) at a particular time.

5.2. Recommendations

Further work is recommended to improve on the results so far obtained for waste management. This may be done by;

(i) Additionally, the skin would be better represented when all three spatial dimensions are included, rather than the one dimension used in this problem solving absorption of Liposome Drug absorption through the skin would involve all three dimensions, and to understand those effects through rigorous modeling would be beneficial.

(ii) Secondly, the model must consider the clearance rate of the absorption of Liposome Drug absorption. The clearance rate determines the quantity of solute absorbed out of the skin into the capillaries. In our models, we did not consider this aspects of the drug absorption.

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REFERENCES

[1] R.L. Bronaugh & R.F. Stewart (2011), "Methods for Percutaneous Absorption Studies. The Flow through Diffusion Cell", *Journal of Pharmaceutical Sciences*, Vol. 74, Pp. 64–67.
[2] K.N. Chao, J.G. Eisley & W.J. Yang (1973), "Heat and Water Migration in Regional Skin and Subcutaneous Tissues", *Bio-MechSypm ASME*, Pp. 69–72.
[3] L.E. Davis (1988), "Cosmetic Formulation Presentation and Prescribing" *Chapter 3 in Veterinary Pharmacology and Therapeutics*, 6th Edition, *Iowa State Press*, Ames.
[4] W.E. Dressler (1999), "Hair Dye Absorption, Bronaugh RL and Maibach HI eds. Percutaneous Absorption: Cosmetic Formulations–Cosmetics–Mechanisms–Methodology", 3rd Edition, New York, Marcel Dekker, *Cosmetic formulations and the Pharmaceutical Sciences*, Vol. 97, Pp. 685–716.
[5] J.S. Gujral, D.M. Proctor, S.H. Su & J.M. Fedoruk (2011), "Skin Adherence Factors for Human Skin", *Risk Analysis*, Vol. 31, Pp. 1271–1280.
[6] J. Hadgraft (1999), "Passive Enhancement Strategies in Topical and Transdermal Drug Delivery", *International Journal of Pharmaceutics*, Vol. 184, Pp. 1–6.
[7] M.A.Khanday & Aasma Rafi (2015), "Variational Finite Element Method to Study the Absorption Rate of Drug at Various Compartments through Transdermal Drug Delivery System, *Alexandria Journal of Medicine*, Vol. 51, No. 3, Pp. 219–223.
[8] P. Morganti, E. Ruocco, R. Wolf & V. Ruocco (2001), "Percutaneous Absorption and Delivery Systems", *Clin Dermatol*, Vol. 19, Pp. 489–501.
[9] Patrick J. Sinko (2004), "Martin's Physical Pharmacy and Pharmaceutical Sciences", 5th Edition, *Pharmacy and Pharmaceutical Sciences*.
[10] M. Rauma, A. Boman & G. Johanson (2013), "Predicting the Absorption of Chemical Vapours", *Advanced Drug Delivery Reviews*, Vol. 65, Pp. 2, Pp. 306–314.
[11] V.P. Saxena & Sharma (2011), "One Dimensional Drug Distribution in Human Dermal Region", *International Journal of Applied Mathematics and Physics*, Vol. 3, No. 1, Pp. 103–118.
[12] V.P. Saxena & D. Arya (1981), "Steady–State Head Distribution in Epidermis, Dermis & Sub Dermal Tissues", *Journal of Theoretical Biology*, Vol. 89, Pp. 423–432.
[13] V.P. Saxena & J.S. Bindra (1987), "Quadratic Shape Function in Variational Finite Element Approach to Heat Distribution Incutaneous and Subcutaneous Tissues", *Indian Journal of Pure and Applied Mathematics*, Vol. 18, Pp. 846–855.
[14] V.P. Saxena (1983), "Temperature Distribution in Human Skin & Sub Dermal Tissue", *Journal of Theoretical Biology*, Vol. 102, Pp. 277–286.
[15] J.E. Scharf, G.T. Johnson, S.C. Harbison, J.D. McCluskey & R.D. Harbison (2008), "Dermal Absorption of a Dilute Aqueous Solution of Malathion", *Journal of Emergencies, Trauma and Shock*, Vol. 1, Pp. 70–73.
[16] R.J. Scheuplein & I.H. Blank (1971), "Permeability of the Skin", *Physiological Reviews*, Vol. 51, Pp. 702–747.
[17] Vineeta Gupta, S.S. Pagey & V.P. Saxena (2012), "Numerical Analysis of Drug Diffusion in Human Dermal Region with Linear Shape Function", *IOSR Journal of Mathematics (IOSR-JM)*, Vol. 4, No. 2, Pp. 31–36.

Augustine Ayanga Mayaka. Mr. Mayaka holds a Bachelor of Education degree in Mathematics and Biology from Kenyatta University in 2008, Kenya. He is undertaking his final project for the requirement of master science in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. He is currently teaching at Mosore Secondary School in Kisii County since 2010 to date. He has keen interest in cosmetic drugs formulation through the human skin and fluid dynamics. Phone number +254-0724040973.

Johana K. Sigey. Prof. Sigey holds a Bachelor of Science degree in mathematics and computer science First Class honors from Jomo Kenyatta University of Agriculture and Technology, Kenya, Master of Science degree in Applied Mathematics from Kenyatta University and a PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. Affiliation: Jomo

Kenyatta University of Agriculture and Technology, (JKUAT), Kenya. He is currently the Director, JKuat, and Kisii CBD. He has been the substantive chairman - Department of Pure and Applied mathematics –Jkuat (January 2007 to July- 2012). He holds the rank of Associate Professor in Applied Mathematics in Pure and Applied Mathematics department – Jkuat since November 2009 to date. He has published 9 papers on heat transfer, MHD and Traffic models in respected journals. Teaching experience: 2000 to date- postgraduate programmes: (JKUAT); Supervised student in Doctor of philosophy: thesis (3 completed, 5 ongoing) ;Supervised student in Masters of science in Applied Mathematics: (13 completed, 8 ongoing).Phone number +254-0722795482.

Jeconia A. Okelo. Dr Okelo holds a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology as well as a Master of Science degree in Mathematics and first class honors in Bachelor of Education, Science; specialized in Mathematics with option in Physics, both from Kenyatta University. I have dependable background in Applied Mathematics in particular fluid dynamics, analyzing the interaction between velocity field, electric field and magnetic field. Has a hand on experience in implementation of curriculum at secondary and university level. He has demonstrated sound leadership skills and has the ability to work on new initiatives as well as facilitating teams to achieve set objectives. He has good analytical, design and problem solving skills. Affiliation: Jomo Kenyatta University of Agriculture and Technology, (JKUAT), Kenya. 2011-To date Deputy Director, School of Open learning and Distance e Learning SODeL Examination, Admission &Records (JKUAT), Senior lecturer Department of Pure and Applied Mathematics and Assistant Supervisor at Jomo Kenyatta University of Agriculture and Technology. Work involves teaching research methods and assisting in supervision of undergraduate and postgraduate students in the area of Applied Mathematics. He has published 10 papers on heat transfer in respected journals. Supervision of postgraduate students; Doctor of philosophy: thesis (3 completed); Masters of Science in applied mathematics: (13 completed, 8 ongoing). Phone number +254-0722971869.

James M. Okwoyo. Dr Okwoyo holds a Bachelor of Education degree in Mathematics and Physics from Moi University, Kenya, Master Science degree in Applied Mathematics from the University of Nairobi and PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. James holds a Bachelor of Education degree in Mathematics and Physics from Moi University, Kenya, Master Science degree in Applied Mathematics from the University of Nairobi and PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. Affiliation: University of Nairobi, Chiromo Campus School of Mathematics P.O. 30197-00100 Nairobi, Kenya. He is currently a lecturer at the University of Nairobi (November 2011 – Present) responsible for carrying out teaching and research duties. He plays a key role in the implementation of University research projects and involved in its publication. He was an assistant lecturer at the University of Nairobi (January 2009 – November 2011). He has published 7 papers on heat transfer in respected journals. Supervision of postgraduate students; Masters of science in applied mathematics: (8 completed and 8 ongoing). Phone number +254-0703602901.

Kang'ethe Giterere. Dr. Kang'ethe Giterere holds a Bachelor of Education science from Kenyatta University, Kenya, Master of Science degree in Applied Mathematics from Kenyatta University, Kenya and PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya. He is currently a lecturer at Jomo Kenyatta University of Agriculture and Technology, Kenya. He has published 9 papers.