Weakly $\ddot{g}$ Closed Sets in Intuitionistic Fuzzy Topological Spaces

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***Abstract*—**In intuitionistic fuzzy topological spaces, closed sets generalization is studied using intuitionistic fuzzy semi-generalized open sets in this paper and it is termed as intuitionistic fuzzy weakly $\ddot{g}$ closed sets.

***Keywords*—**Intuitionistic Fuzzy Weakly $\ddot{g}$ Closed Sets, Intuitionistic Fuzzy Set;

***Abbreviations*—**Fuzzy Set (FS); Intuitionistic Fuzzy Closed Set (IFCS); Intuitionistic Fuzzy Open Set (IFOS); Intuitionistic Fuzzy Topological Spaces (IFTS); Intuitionistic Fuzzy Topology (IFT).

I. INTRODUCTION

Spaces and shapes mathematical study corresponds to topology. Out of set theory and geometry, topology is developed as a field of study, where transformation, dimension and space concepts are analyzed. Generalization closed sets study us initiated by Levine [8] in 1970. In 1965, Zadeh [14] introduced FS concepts and in 1968, Chang [3] introduced a fuzzy topology. Fuzzy semi-open sets are investigated as well as introduced by Ganguly and Saha [6] in 1986. In fuzzy topological spaces, generalized closed sets concepts are introduced by Sundaram and Balasubramanian [2] in 1997. In 1986, IFS concepts are proposed by Atanassov [1], where fuzzy set is added with non-membership degree using fuzzy sets notion. Intuitionistic fuzzy closed sets, open sets and topological spaces concepts are introduced by Coker [4] in 1997. In intuitionistic fuzzy topological spaces, generalized closed sets are introduced by Rekha Chaturvedi and Thakur [13]. In IFTS, closed sets generalization development are contributed by various researchers [5,7,10].

Generalized closed sets forms a strong tool in intuitionistic fuzzy topological spaces characterization satisfying weak separation axioms. Rajarajeswari and Krishna Moorthy [9], Rajarajeswari and L. Senthil Kumar [10] and Thakur and Rekha Chaturvedi [12] have introduced IFCS which are generalized weakly, intuitionistic fuzzy regular closed sets which are generalized weakly and intuitionistic fuzzy regular generalized closed sets respectively. In this chapter, as generalization of closed sets, in IFTS's new class namely intuitionistic fuzzy weakly $\ddot{g}$ closed sets is studied in this paper.

II. PRELIMINARIES

In this paper, IFS is represented as X or (X, τ). For X's subset A, closure is represented as cl(A), interior is represented as int(A), A's complement is represented as A^C. In sequel, some basic definitions which are utilized are recalled here.

$A = \{ \langle x, \mu A(x), \nu A(x)\rangle / x \in X \}$ is replaced by notation $A = \langle x, \mu A, \nu A\rangle$, $A = \langle x, (A/\mu A, B/\mu B), (A/\nu A, B/\nu B)\rangle$ is replaced by notation $A = \langle x, (\mu A, \mu B), (\nu A, \nu B)\rangle$ for simplification. In X, empty set is given by IFS $0\sim = \{<x, 0, 1> / x \in X\}$, whole set is given by $1\sim = \{<x, 1, 0> / x \in X\}$.

2.1. Definition 2.1.

Assume non-empty set as X [1]. In X, IFS A is an object with $A = \{<x, \mu_A(x), \nu_A(x)> / x \in X\}$ form, where, every element $x \in X$ in set A, membership degree is represented using function $\mu_A : X \to [0, 1]$ and it is named as μ_A (x), non-membership degree is represented as $\nu_A : X \to [0, 1]$ and it is named as ν_A (x), for every $x \in X$, $0 \le \mu_A (x)+\nu_A (x) \le 1$. In X, all IFS is represented as IFS (X).

2.2. Definition 2.2.

Assume A, B are IFSs and are given by $B = \{<x, \mu_B(x), \nu_B(x) > / x \in X\}$, $A =\{<x, \mu_A (x), \nu_A (x) > / x \in X$,. Then [1]

1. for every $x \in X$, if and only if $\nu_A(x) \ge \nu_B(x)$ and $\mu_A (x) \le \mu_B(x)$, $A \subseteq B$
2. if $B \subseteq A$, $A \subseteq B$ then $A = B$
3. $A^C = \{<x, \mu_A(x), \nu_A(x), > / x \in X\}$,
4. $A \cap B = \{<x, \nu_A(x) \vee \nu_B(x), \mu_A(x) \wedge \mu_B(x), > / x \in X\}$,
5. $A \cup B = \{<x, \nu_A(x) \wedge \nu_B(x), \mu_A(x) \vee \mu_B(x), > / x \in X\}$.

2.3. Definition 2.3.

In X, IFSs family τ is an IFS. Following axioms are satisfied by this [4].

1. $0\sim, 1\sim \in \tau$,
2. $G1 \cap G2 \in \tau$ for every $G1, G2 \in \tau$,
3. $\cup Gi \in \tau$ for each family $\{Gi / i \in J\} \subseteq \tau$.

In X, pair (X, τ) is termed as IFTS and in τ, any IFS is termed as IFOS. In an IFTS (X, τ), A^C complement is termed as IFTS.

2.4. Definition 2.4.

In X, assume (X, τ) as IFTS and A=<x, μ_A, ν_A> as IFS. Then [4]

1. int(A) = ∪{ G ⊆ A, G / G is an IFOS in X },
2. cl(A) = ∩{ A ⊆ K and K / K is an IFCS in X },
3. $cl(A^C) = (int(A))^C$,
4. $int(A^C) = (cl(A))^C$.

2.5. Result 2.5.

In IFTS (X, τ), any two IFS are assumed as A and B. Then cl(A ∪ B)=cl(A) ∪ cl(B).

2.6. Definition 2.6.

In an IFTS (X, τ), IFS A = <x, μ_A, ν_A> is termed as [7]

1. if int(cl(A)) ⊆ A, then it is termed as IFSCS
2. if A ⊆ cl(int(A)), then it is termed as IFSOS
3. if cl(int(cl(A))) ⊆ A, then it is termed as intuitionistic fuzzy α -closed set (IF α CS)
4. if A ⊆ int(cl(int(A))), then it is termed as intuitionistic fuzzy α -open set (IF α OS)
5. if cl(int(A)) = A, then it is termed as intuitionistic fuzzy regular closed set (IFRCS)
6. if A = int(cl(A)), then it is termed as intuitionistic fuzzy regular open set (IFROS)
7. if cl(A) ⊆ U, then it is termed as intuitionistic fuzzy generalized closed set (IFGCS) whenever A⊆U, in X, U is an IFOS. IFGCS complement is IFGOS,
8. if cl(A) ⊆ U, then it is termed as intuitionistic fuzzy regular generalized closed set (IFRGCS) whenever A ⊆ U, in X, U is an IFROS. IFRGCS complement is IFRGOS.

2.7. Definition 2.7.

In IFTS (X, τ), IFS A = <x, μ_A, ν_A> is termed as

1. if cl(A) ⊆ U, then it is termed as IFGCS, whenever A ⊆ U, in X, U is IFOS. IFGCS complement is IFGOS [14],
2. if scl(A) ⊆ U, then it is termed as IFSGCS whenever A ⊆ U, in X, U is IFSOS. IFSGCS complement is an IFSGOS [11],
3. if cl(int(A)) ⊆ U, then it is termed as intuitionistic fuzzy weakly generalized closed set (IFWGCS) whenever A ⊆ U, in X, U is IFOS. IFWGCS complement is IFWGOS [9],
4. if scl(A) ⊆ U, then it is termed as intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS) whenever A ⊆ U and in X, U is IFROS. IFRWGCS complement is an IFRWGOS [10, 11].

III. Intuitionistic Fuzzy Weakly g̈ Closed Sets

3.1. Definition 3.1.

In IFTS (X, τ), if cl(A) ⊆ U, then IFS A is termed as intuitionistic fuzzy $\ddot{g}$ closed set(IF$\ddot{g}$CS) whenever A ⊆ U and in (X, τ), U is an IFSGOS.

An IFTS (X, τ)'s all IF$\ddot{g}$CSs family is represented as IF$\ddot{g}$CS (X). In (X, τ), IF$\ddot{g}$CS complement is IF$\ddot{g}$OS.

3.2. Theorem 3.2.

In (X, τ), each IFCS is IF$\ddot{g}$CS, but not conversely.

Proof:

Assume A as an IFCS and A ⊆ U where, in (X, τ) and U is IFSGOS. Then cl(A) = A ⊆ U as per hypothesis. Hence, in (X, τ), A is IF $\ddot{g}$CS.

Example 3.3.

Assume G = <x, (0.5, 0.6), (0.5, 0.4)> and X = {a, b}. Then on X, τ ={0~, G, 1~} is an IFT and IFS A = <x, (0.4, 0.4), (0.6, 0.6) > is IF$\ddot{g}$ CS, but in (X, τ), this is not IFCS.

3.3. Theorem 3.4.

In (X, τ), each IFRCS is IF$\ddot{g}$CS, but not conversely.

Proof

Since every IFRCS is IFCS and as mentioned in Theorem 3.3., in X, A is an IF$\ddot{g}$CS.

Example 3.5.

Take G = <x, (0.5, 0.6), (0.5, 0.4)> and X = {a, b}. Then on X, τ ={0~, G, 1~} is an IFT and IFS A = <x, (0.4, 0.4), (0.6, 0.6)> is IF$\ddot{g}$ CS but in (X, τ), it is not IFRCS.

3.4. Theorem 3.6.

In (X, τ), each IF$\ddot{g}$CS is IFGCS, but not conversely.

Proof:

Assume A ⊆ U and in (X, τ), U as IFOS. Since each IFOS is IFSGOS and cl(A) ⊆ U as per hypothesis. Hence in (X, τ) A is IFGCS.

Example 3.7.

Assume G = <x, (0.8, 0.8), (0.2, 0.1)> and X = {a, b}. Then on X, τ ={0~, G, 1~} is IFT, IFS A = <x, (0.8, 0.7), (0.1, 0.3)> is IFGCS but in (X, τ), it is not IF$\ddot{g}$CS.

3.5. Theorem 3.8.

In (X, τ), each IF$\ddot{g}$ CS is IFRGCS, but not conversely.

Proof:

Assume A ⊆ U and in (X, τ), U as IFROS. Since each IFROS is IFSGOS and cl(A) ⊆ U as per hypothesis. Hence in (X, τ), A is IFRGCS.

Example 3.9.

Assume G = <x, (0.8, 0.8), (0.2, 0.1)> and X = {a, b}. Then on X, τ={0~, G, 1~} is IFT, IFS A = <x, (0.9, 0.7), (0.1, 0.3)> is IFRGCS but in (X, τ), it is not IF$\ddot{g}$CS.

3.6. Theorem 3.10.

In (X, τ), each IF$\ddot{g}$CS is IFG α CS, but not conversely.

Proof:

Assume A ⊆ U and in (X, τ), U as IF α OS. Since each IF α OS is IFSGOS and αcl(A) ⊆ cl(A) ⊆ U as per hypothesis. Hence in (X, τ), A is IFG α CS.

Example 3.11.

Assume G = <x, (0.3, 0.2), (0.7, 0.7)> and X = {a, b}. Then on X, τ ={0~, G, 1~} is IFT, IFS A = <x, (0.6, 0.6), (0.3, 0.4)> is IFG α CS but in (X, τ), this is not IF$\ddot{g}$ CS.

3.7. Theorem 3.12.

In (X, τ), each IF$\ddot{g}$ CS is IFWCS, but not conversely.

Proof:

Assume A ⊆ U and in (X, τ), U as IFSOS. Since every IFSOS is IFSGOS and by hypothesis, cl(A) ⊆ U. Hence in (X, τ), A is IFWCS.

Example 3.13.

Assume G = <x, (0.7, 0.7), (0.3, 0.3)> and X = {a, b}. Then on X, τ ={0~, G, 1~} is IFT, IFS A = <x, (0.4, 0.4), (0.6, 0.6)> is IFWCS but in (X, τ), this is not IF$\ddot{g}$ CS.

3.8. Theorem 3.14.

In IFTS (X, τ), assume A, B are two IF$\ddot{g}$ CSs then in X, A ∪ B is IF$\ddot{g}$ CS.

Proof:

In (X, τ), assume U as IFSGOS such that A ∪ B ⊆ U. Since A, B are IF g¨ CSs, cl(B) ⊆ U and cl(A) ⊆ U. Therefore, using result 1.2.6, cl(A) ∪ cl(B) = cl(A ∪ B) ⊆ U. Hence in (X, τ), A ∪ B is IF g¨ CS.

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