An Iterative Method for Solving Unsymmetric System of Fuzzy Linear Equations

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***Abstract*—**Recently, the solution of Fuzzy Linear System of Equations (FLSE) has been studied by many authors in literature. In many cases, some suitable methods of Crisp Linear System of Equations (CLSE) are extended to compute the solution of FLSE. Here, we suggest a new iterative method for solving FLSE based on the Hermitian and Skew-Hermitian Splitting (HSS) method which HSS is a recently proposed iterative method for CLSE, where the coefficient matrix is a crisp unsymmetric matrix. In HSS method, the coefficient matrix is into two Hermitian and Skew-Hermitian matrices and the iterative solutions are obtained by solving two well-conditioned systems of equations instead of the original problem, sequentially. Our suggested method is converged to the solution for all kinds of FLSE problems with triangular or rectangular fuzzy numbers. To examine this method, numerical results of some problems are illustrated at the end of this paper.

***Keywords*—**FLSE; Fuzzy Linear System; HSS; Iterative Methods; Triangular Fuzzy Numbers.

***Abbreviations*—**Conjugate Gradient (CG); Crisp Linear System of Equations (CLSE); Fuzzy Linear System of Equations (FLSE); Generalized Minimal Residual (GMRES); Hermitian and Skew-Hermitian Splitting (HSS).

I. INTRODUCTION

THERE are many applications for the solution of linear system of equations. Due to the importance of these solutions for various problems, scientists have paid especial attention to solve such problems by iterative methods as ease and fast as possible. So, it is important to study more about variety of efficient techniques for solving linear system of equations. There are many iterative methods such as GMRES, different CG variants, etc., which have been proposed for the solution of such systems [Datta, 1995; Saad, 1996; Barrett et al., 2000]. Many authors have studied different properties of current methods in literature. For general information about such methods refer to [Friedman et al., 1998; Barrett et al., 2000].

In many cases, at least some of the system's parameters are represented by fuzzy rather than crisp numbers to fit more the problem with its nature that this problem is extracted from discritization of some PDEs or other applicable equations. The solution of such fuzzy linear systems is applicable in different areas such as economic, engineering, physics etc. Therefore, it is essential to develop solving methods, which appropriately treat fuzzy linear systems, for finding suitable solutions.

For the first time, the concept of fuzzy numbers and associated arithmetic operations was proposed by Zadeh. Next, a general model for solving an $n \times n$ FLSE, where the coefficient matrix is crisp and the right hand side is a fuzzy vector, was proposed by Friedman et al., (1998). They extended their work by considering duality for fuzzy linear systems in Friedman et al., (2000). Wang et al., (2001) proposed some iterative methods for the solution of such systems. After that, many scientists have focused on FLSE not only because of the variety of its applications even in other majors, but also because it may be possible to extend some suggested methods, used for computing the solution of ordinary linear systems, for fuzzy problems under some especial circumstances. So, in literature, there are many techniques such as Jacobi, Gauss-Seidel, SOR and steepest decent as iterative methods [Wang et al., 2001; Allahviranloo, 2004, 2005; Abbasbandy et al., 2005; Dehghan & Hashemi, 2006; Abbasbandy & Jafarian, 2006], which extended from ordinary problems to fuzzy systems. Abbasbandi et al., (2005) proposed a Conjugate Gradient method for symmetric fuzzy linear systems, Allahviranloo (2005A) suggested an Adomian decomposition method to compute the solution of such system. Some direct methods like LU decomposition in Abbasbandy et al., (2006), as well as block iterative methods [Wang & Zheng, 2007] and lots of other approaches [Nasseri & Khorramizadeh, 2007; Allahviranloo & Ghanbari, 2012; Senthilkumar & Rajendran 2013; Allahviranloo et al., 2013; Najafi & Edalatpanad, 2013; Najafi et al., 2013] for solving FLSE have been proposed. The works of Saberi and his colleagues [Najafi & Edalatpanad, 2013; Najafi et al., 2013] and Allahviranloo and his colleagues [Allahviranloo & Ghanbari, 2012; Allahviranloo et al., 2013] are some recently proposed methods for the solution of such problems.

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Bai and his colleagues (2003; 2005) developed some new iterative methods for the solution of CLSE. In these works, they split the coefficient matrix into two Hermitian and Skew-Hermitian matrices. Then, two different linear systems are generated. So, the solution of original system is computed by solving these two systems. In this work, we extend this idea for the solution of FLSE. Thus, we deal with the problem

$$A\tilde{x} = \tilde{y} \tag{1}$$

where $A \in R^{n\times n}$ and $\tilde{y}$ is a fuzzy vector. More explanation about this method is in section 3.

This paper is organized as follows. In section 2, we discuss some basic definitions and results on fuzzy numbers and FLSE. The new proposed method for solving such fuzzy systems is discussed in section 3. Numerical tests and conclusion are drawn in the next sections.

II. Basic Concepts and Definitions

Here, some primary definitions and notes, which are required in this study, have been indicated from [Friedman et al., 1998; 2000; Allahviranloo, 2004; 2005; 2005A; Abbasbandy et al., 2005; 2006].

Definition 1: The r-level set of a fuzzy set $\tilde{u}$ is defined as an ordinary set $[\tilde{u}]_r$ of which the degree of membership function exceeds the level r, i.e.

$$\tilde{u}_r = [\tilde{u}]_r = \{ x \in R \mid \tilde{u}(x) \ge r \quad , r \in [0,1] \} \tag{2}$$

Definition 2: A fuzzy set $\tilde{u}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

- $\tilde{u}$ is convex i.e.

$$\mu_{\tilde{u}}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{u}}(x_1), \mu_{\tilde{u}}(x_2)) \forall x_1, x_2 \in R, \forall \lambda \in [0,1] \tag{3}$$

- $\tilde{u}$ is normal i.e. $\exists x_0 \in R$ such that $\mu_{\tilde{u}}(x_0) = 1$.
- $\mu_{\tilde{u}}$ is piecewise continuous.

Definition 3: A fuzzy number $\tilde{u}$ in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r)$, $0 \le r \le 1$, that satisfies the following requirement:

- $\underline{u}(r)$ is a bounded monotonically increasing left continuous function;
- $\overline{u}(r)$ is a bounded monotonically decreasing left continuous function;
- $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

Definition 4: The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows, see Friedman et al., (1998).

For arbitrary $\tilde{u} = (\underline{u}, \overline{u})$, $\tilde{v} = (\underline{v}, \overline{v})$ and $k \in R$, the addition and the scalar multiplication are defined as follows:

- $\underline{u} = \overline{v}$ iff $\underline{u}(r) = \underline{v}(r)$ and $\overline{u}(r) = \overline{v}(r)$,
- $\tilde{u} \pm \tilde{v})(r) = \left(\left(\underline{u}(r) \pm \underline{v}(r)\right), (\overline{u}(r))\right)$,
- $k\tilde{u} = \begin{matrix} (k\underline{u}, k\overline{u})(r), k \ge 0, \\ (k\overline{u}, k\underline{u})(r), k < 0. \end{matrix}$

Remark 1: A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \le r \le 1$.

Definition 5: The triangular fuzzy number $\tilde{u} = (u_1, u_2, u_3)$ is a fuzzy set where the membership function is as

$$\tilde{u}(x) = \begin{cases} \dfrac{x-u_1}{u_2-u_1}, & u_1 \le x \le u_2, \\ \dfrac{u_3-x}{u_3-u_2}, & u_2 \le x \le u_3, \\ 0, & Otherwise; \end{cases} \tag{4}$$

and its parametric form is

$$\tilde{u} = (\underline{u}(r), \overline{u}(r)),\ \underline{u}(r) = (u_2 - u_1)r + u_1, \quad \overline{u}(r) = u_3 - (u_3 - u_2)r. \tag{5}$$

Definition 6: A triangular fuzzy number $\tilde{u}$ is said to be non-negative fuzzy number if and only if $\tilde{u}(x) = 0, \forall x < 0$.

For solving a $n \times n$ fuzzy linear system

$$A\tilde{x} = \tilde{b} \tag{6}$$

with a crisp square matrix A and a triangular fuzzy vector $\tilde{b}$ different iterative methods have been proposed.

The ith row of fuzzy linear system with the solution $\tilde{x} = (\tilde{x}_1, , , , , \tilde{x}_n)^T$, $\tilde{x}_i = \left(\bar{x}_i(r), \underline{x_i}(r)\right)$, $i = 1, \dots, n$ is as

$$\underline{\sum_{j=1}^{n} a_{i,j} x_j} = \sum_{j=1}^{n} \underline{a_{i,j} x_j} = \underline{b_i},$$
$$\overline{\sum_{j=1}^{n} a_{i,j} x_j} = \sum_{j=1}^{n} \overline{a_{i,j} x_j} = \overline{b_i}, \quad i = 1, \dots, n$$

From the above, it is extracted that

$$\underline{a_{i,1} x_1} + \cdots + \underline{a_{i,n} x_n} = \underline{b_i}(r)$$
$$\overline{a_{i,1} x_1} + \cdots + \overline{a_{i,n} x_n} = \overline{b_i}(r)$$

Based on the idea of Friedman et al., the system (6) is converted into a $2n \times 2n$ crisp function linear system

$$EX = Y \tag{7}$$

where matrix $S = (S_{ij})$ is obtained as follows:

$$\begin{cases} a_{ij} \ge 0 \quad \Rightarrow \quad e_{ij} = a_{ij}, \quad e_{i+n,j+n} = a_{ij}, \\ a_{ij} < 0 \quad \Rightarrow \quad e_{i,j+n} = -a_{ij}, \quad e_{i+n,j} = -a_{ij}, \end{cases} \tag{8}$$

and each e_{ij} which is not determined by (8) is zero, $Y = \left(\underline{b_1}, \dots, \underline{b_n}, -\overline{b_1}, \dots, -\overline{b_n}\right)^T$ and $X = \left(\underline{x_1}, \dots, \underline{x_n}, -\overline{x_1}, \dots, -\overline{x_n}\right)^T$.

The matrix E is determined as a symmetric block matrix $E = \begin{pmatrix} F & G \\ G & F \end{pmatrix}$ where $f_{i,j} = e_{i,j}$ and $g_{i,j} = e_{i+n,j}$.

An approximate solution of (7) that is often used is the least square solution of (7), defined as a vector X which minimizes the Euclidean norm of $(Y - EX)$.

Definition 7: Let $x = \left\{\left(\underline{x_j}(r), \overline{x_j}(r)\right), 1 \le j \le n\right\}$ denote the solution of $EX = Y$. The triangular fuzzy vector $\tilde{U} = \left(\underline{u_j}(r), \overline{u_j}(r)\right), 1 \le j \le n\}$ defined by $\underline{u_j}(r) = min\{\underline{x_j}(r), \overline{x_j}(r), \underline{x_j}(1)$ and $\overline{u_j}(r) = max\{\underline{x_j}(r), \overline{x_j}(r), \underline{x_j}(1)$ is called the fuzzy solution of $EX = Y$. If $\forall j: \underline{u_j} = \underline{x_j}$ and $\overline{u_j} = \overline{x_j}$, U is called a strong fuzzy solution.

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III. Solving Fuzzy Linear Systems

There are various iterative methods for solving different crisp linear system of equations [Datta, 1995; Saad, 1996; Barrett et al., 2000]. In many papers, to solve the crisp system

$$Bu = f \tag{9}$$

where $B \in R^{n\times n}$ and $u, f \in R^n$, the coefficient matrix B has been split as $B = M - N$ and the original problem is replaced by iterative method

$$u^{(k+1)} = M^{-1}Nu^{(k+1)} + M^{-1}f \text{, for } k = 0,1,2,\dots . \tag{10}$$

It proved that (10) is converges if $\|M^{-1}N\| < 1$, where is the induced Euclidean norm for the square matrices (see [Datta, 1995; Saad, 1996]).

Recently Bai et al., (2003; 2005) suggested some new iterative methods based on similar splitting pattern. They supposed to split the coefficient matrix B into two different matrices H and S so that $B = H + S$, where $H = (B + B^T)/2$ and $S = (B - B^T)/2$ with B^T as the transpose of B. In fact, B and S are Hermitian and Skew-Hermitian matrices, respectively.

In this way, the original problem $Bu = f$ is transformed into two systems of equations

$$i.\ (\alpha I + H)u = (\alpha I - S)u + b,$$
$$ii.\ (\alpha I + S)u = (\alpha I - H)u + b,$$

where α is a given positive constant. Suppose $u^{(0)}$ is the initial guess. To compute the solution of original system, at first, an approximation of the first equation is obtained by an appropriate iterative method. Then, the computed solution of first equation is considered as the initial vector of the second equation. By applying the iterative method on the second system, the next approximation of original problem is resulted. This procedure is continued until the computed approximation satisfied.

So, the iterative sequence $\{u^{(k)}\}$ for integer $k \geq 0$ is computed by

$$\begin{cases} \text{i.} & (\alpha I + H)u^{(k+\frac{1}{2})} = (\alpha I - S)u^{(k)} + b, \\ \text{ii.} & (\alpha I + S)u^{(k+1)} = (\alpha I - H)u^{(k+\frac{1}{2})} + b, \end{cases} \tag{11}$$

until $\{u^{(k)}\}$ converges. If we set

$$M = \frac{1}{2\alpha}(\alpha I + H)(\alpha I + S),\quad N = \frac{1}{2\alpha}(\alpha I - H)(\alpha I - S),$$

then the matrix splitting $B = M - N$, as the key point of the above iterative algorithm, yields so that we have

$$Mu = Nu + b.$$

Now, we use this technique to find the solution of FLSE (1). For this aim, it is more convenient to apply HSS algorithm on the parametric crisp linear system (7). So, if $X^{(0)}$ is the initial vector for the solution of $EX = Y$, the iterative sequence $\{X^{(k)}\}$ is obtained by

$$\begin{cases} \text{i.} & (\alpha I + H)X^{(k+\frac{1}{2})} = (\alpha I - S)X^{(k)} + Y, \\ \text{ii.} & (\alpha I + S)X^{(k+1)} = (\alpha I - H)X^{(k+\frac{1}{2})} + Y, \end{cases}$$

where H and S are Hermitian and Skew-Hermitian matrices of E, respectively. This procedure is continued until $\{X^{(k)}\}$ converges.

To know more about HSS algorithm, the following two theorems are helpful.

Theorem 1: [Bai et al., 2003] Let $E \in R^{n\times n}$ and $E = M_i - N_i$ $(i = 1,2)$ be two matrix splitting of matrix E, and $X^{(0)}$ be a given initial vector. Suppose $\{X^{(k)}\}$, $k \geq 0$ is a two-step iteration sequence defined by

$$\begin{cases} \text{i.} & M_1X^{(k+\frac{1}{2})} = N_1X^{(k)} + b, \\ \text{ii.} & M_2X^{(k+1)} = N_2X^{(k+\frac{1}{2})} + b, \end{cases}$$

then

$$X^{(k+1)} = M_2^{-1}N_2M_1^{-1}N_1X^{(k)} + M_2^{-1}\left(I - N_2M_1^{-1}\right)b,\ k \geq 0.$$

Moreover, if the spectral radius $\rho(M_2^{-1}N_2M_1^{-1}N_1)$ is less than 1, then the iterative sequence $\{X^{(k)}\}$ converges to the unique solution of (7).

Theorem 2: [Bai et al., 2003] Let $E \in R^{n\times n}$ be a positive definite matrix and $H = (E - E^T)/2$ and $S = (E - E^T)/2$ and α be a given positive constant. Then the iteration matrix $M(\alpha)$ of the HSS iteration is given by

$$M(\alpha) = (\alpha I + S)^{-1}(\alpha I - H)(\alpha I + H)^{-1}(\alpha I - S)$$

and its spectral radius $\rho(M(\alpha))$ is bounded by

$$\sigma(\alpha) \equiv \max_{\lambda_i \in \lambda(H)} \left| \frac{\alpha - \lambda_i}{\alpha - \lambda_i} \right|,$$

where $\lambda(H)$ is the spectral set of the matrix H. Therefore, it holds that

$$\rho(M(\alpha)) \leq \sigma(\alpha) < 1, \quad \forall \alpha > 0;$$

and hence, HSS iteration converges to the unique solution of (6).

IV. Numerical Examples

To examine, the convergence of our suggested method for solving FLSE, the following fuzzy systems are solved by this method. Here, we suppose $X_0 = 0$ is the initial guess and $\alpha = 0$ is the constant factor for both problems.

Example 1: Consider the 2×2 fuzzy linear system

$$\tilde{x}_1 - 2\tilde{x}_2 = (r, 2 - r),$$
$$\tilde{x}_1 + 3\tilde{x}_2 = (4 + r, 7 - 2r).$$

From this fuzzy system we have

$$E = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix},\quad Y = \begin{pmatrix} r \\ 4 + r \\ r - 2 \\ 2r - 7 \end{pmatrix}$$

By applying the above method on this system, the exact solution

$X = \left(1.375+0.625 \quad 0.875+0.125r \quad 0.875r-2.785 \quad 0.375r-1.375\right)^T$,

by $eps = 1e-6$ is estimated after 25 iterations. For this example, we have

$$H = \frac{1}{2}\left(E+E^T\right) = \begin{pmatrix} 1 & 0.5 & 0 & 0.5 \\ 0.5 & 3 & 0.5 & 0 \\ 0 & 0.5 & 1 & 0.5 \\ 0.5 & 0 & 0.5 & 3 \end{pmatrix}$$

and

$$S = \frac{1}{2}\left(E-E^T\right) = \begin{pmatrix} 0 & -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 & 0 \\ 0 & 0.5 & 0 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \end{pmatrix}.$$

In this example, we find the solution of a 2×2 FLSE by solving two parametric problems of order 4. The next example is to show the performance of this algorithm on a fuzzy linear system with 6 fuzzy variables.

Example 2: Now, the following 6×6 fuzzy linear system is considered

$$\begin{pmatrix} 4 & 0 & 1 & 0.5 & -1 & 0 \\ -1 & 4 & 0 & 1 & 0.5 & -1 \\ 0 & -1 & 4 & 0 & 1 & 0.5 \\ 0.21 & 0 & -1 & 4 & 0 & 1 \\ & 0.21 & 0 & -1 & 4 & 0 \\ & & 0.21 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} \left(\underline{x}_1, \overline{x}_1\right) \\ \left(\underline{x}_2, \overline{x}_2\right) \\ \vdots \\ \left(\underline{x}_6, \overline{x}_6\right) \end{pmatrix} = \begin{pmatrix} \left(r+1, 3-r\right) \\ \left(r+2, 4-r\right) \\ \vdots \\ \left(r+6, 8-r\right) \end{pmatrix}.$$

Here, the 12×12 coefficient matrix E and right hand side vector Y are generated based on (7). We apply HSS algorithm to solve this problem. This method finds the solution of this system after 34 iterations for $eps = 1e-6$ while the exact solution is

$$\tilde{x} = \begin{pmatrix} \left(\underline{x}_1, \overline{x}_1\right) \\ \left(\underline{x}_2, \overline{x}_2\right) \\ \vdots \\ \left(\underline{x}_6, \overline{x}_6\right) \end{pmatrix} = \begin{pmatrix} \left(0.1434r+0.5043\ ,\ 0.7912-0.1434r\right) \\ \left(0.0983r+0.9444\ ,\ 1.1410-0.0983r\right) \\ \left(0.1493r+0.4661\ ,\ 0.7648-0.1493r\right) \\ \left(0.1574r+0.6854\ ,\ 1.0001-0.1574r\right) \\ \left(0.1983r+1.4295\ ,\ 1.8261-0.1983r\right) \\ \left(0.1926r+1.9222\ ,\ 2.3074-0.1926r\right) \end{pmatrix}.$$

V. Conclusion

To find the solution of a FLSE, we used HSS algorithm on the extended parametric system of equations extracted from original system. By this approach, the original parametric system is replaced by two well-conditioned systems and the iterative approximations are obtained in two steps. From the first equation, a vector is computed and this vector is considered as the initial vector of the second system. In fact, the approximations of this method are the vectors computed by the second equation. As the numerical tests show, our suggested method is practical for solving fuzzy linear system of equations with a crisp square coefficients matrix and a fuzzy right hand side vector. So, we recommend this method for solving such problems.

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