The Evaluation of Automobile in Germany via Globalization Grey Relational Grade

Kun-Li Wen*, Keng-Hao Chang** & Yi-Cheng Shen***

*Professor, Department of Electrical Engineering, Chienkuo Technology University, Changhua, TAIWAN.
E-Mail: klw{at}ctu{dot}edu{dot}tw
**Master student, Department of Electrical Engineering, Chienkuo Technology University, Changhua, TAIWAN.
E-Mail: keng.hao{at}hotmail{dot}com
***Junior student, Department of Electrical Engineering, Chienkuo Technology University, Changhua, TAIWAN.
E-Mail: otisa1224{at}gmail{dot}com

***Abstract*—**As we all know, Germany is not only the birthplace of the modern automobile but also the oldest automobile production country of the world. The top five automakers, including Volkswagen, BMW, Mercedes-Benz, Audi and Porsche, have sold cars around the world. In view of this, the paper applies five kinds of cardinal globalization grey relational grade in the grey system theory, which is regarded as a soft computing analysis method, to rank the five automobile makers. This study, hopefully, can provide criteria whereby consumers can select their favorite brand. The paper draws information from the official websites of the five depots. Eight factors, including liter, horse power, maximum torque, compress ratio, price, 0-100 km/h acceleration time, weight and fuel consumption, are identified as influencing items. The cardinal globalization grey relational grade is subject to calculation to obtain the rank of the five automakers. The data analysis shows the five automobile makers do not significantly differ from one another as to quality. This suggests buyers can safely select any brand to meet their needs.

***Keywords*—**Automobiles; Cardinal; Germany; Globalization Grey Relational Grade; Rank.

***Abbreviations*—**Analytic Hierarchy Process (AHP); Grey Weighting Model for n Variables (GM(0,N)); Localization Grey Relational Grade (LGRA); Multiple Criteria Decision Making (MCDM).

I. INTRODUCTION

SINCE Karl Benz invented the first automobile in 1886, Germany's auto industry has undergone 120 years of development. Three stages of development can be identified between 1886 and 1960: 1) the invention and experiment of the automobile from 1886 to 1910, 2) the technical perfection from 1911 to 1940 and 3) rapid development from 1941 to 1960. Germany has become Europe's largest automobile manufacturer, and it is also one of the most important global automotive production bases [Tsai, 1]. Since 2005, the average turnover of Germany automobile manufacturers and suppliers are more than 300 billion euros per year. In the first half of 2013, Europe's top five producing countries, led by Germany, has five brands of automobiles, including VW, BMW, Mercedes-Bens, Audi and Porsche [2, 3, 4, 5, 6]. The sales figure reached to 1,623,000 vehicles. Germany-made automobiles were marketed domestically and internationally as well. In addition, Germany has quite a lot of experienced and professional automotive technicians and engineers. The education training track in Germany has provided outstanding industry-oriented training. There are approximately 80,000 technical personnel receiving specialized technical training each year. The world's leading automobile manufacturers have set up factories in Germany [7]. Therefore, the paper implements the cardinal globalization grey relational grade in the grey system theory to conduct relative comparisons of the automotive performances of the five brands. The results can serve as guiding principles to consumers in their automobile purchase [Wen, 8].

There have been over the past few years some relevant publications on automobile purchase. Some used fuzzy MCDM to study automobiles buying [Tang, 9; Helili, 10]; others applied AHP to investigate car buying; still others were concerned about managing automobile purchasing [Jian et al., 11; Chan et al., 12; Lai, 13; Chiang, 14; Pan, 15; Shi & Hu, 16]. There were also publications using the grey system theory, some of which applied GM(0,N), MCDM and LGRA [Lu & Wu, 17; Hu, 18; Sakthivel et al., 19].

The paper proposed to use the cardinal globalization grey relational grade of the grey system theory. The data for analysis were obtained from the official websites of the five automobile companies, Eight factors, including Liter, Horse power, Maximum Torque, Compress ratio, Price, 0-100 km/h Acceleration time, Weight and Fuel consumption serve as the influencing factors. The preferable choices of five brands are analyzed through mathematical calculation and the result of study can help consumers in their purchase of automobiles.

Section 2 introduces the mathematical model. Section 3 gives the five brands of Germany's automobiles to verify the mathematical method mentioned above. Finally, conclusion and future prospects are discussed and recommendation is provided for future research.

II. MATHEMATICS MODEL

The grey system theory was proposed by Professor Deng in 1989. The grey relational grade is the most important topic in the grey system theory. The main function of the grey relational grade is to conduct the measurement between two discrete sequences. The whole analysis steps are shown below [Wen, 20].

2.1. Basic Concept

2.1.1. Factor Space

Assume $\{P(X)\}$ is the theme, Q is a relationship, and $\{P(X);Q\}$ has four conditions, including existence of factor, accountability of factor, expansion of factor, and factor of independent, then $\{P(X);Q\}$ is called factor space.

2.1.2. Comparison

For sequence is

$$x_i = (x_1(k),\cdot,\cdot\cdot,x_i(k)) \in X;\quad k = 1,2,3,\cdots,n \in N,\ i = 0,1,2,\cdots,m \in I, \quad X \text{ is Universal set} \tag{1}$$

If equation (1) satisfies three conditions, namely non-dimension, scaling and polarization, then x_i is called the comparison of sequence.

2.1.3. Measurement

If a space satisfies the conditions of factor space and ccomparison, then it is called grey relational space, and is written as $\{P(X);\Gamma\}$, where $\{P(X)\}$ is the theme, and Γ is the measurement.

2.2. Grade Relational Grade

If there exists a function $\gamma(x_i,x_j) \in \Gamma$ which satisfies four axioms, including normality, duality symmetric, wholeness and closeness, then $\gamma(x_i,x_j)$ is called grey relational grade.

Based on equation (1), we can write the sequences, as shown in equation (2).

$$\begin{aligned} x_0 &= (x_0(1),x_0(2),\cdot,\cdot\cdot,x_0(k)) \\ x_1 &= (x_1(1),x_1(2),\cdot,\cdot\cdot,x_1(k)) \\ x_2 &= (x_2(1),x_2(2),\cdot,\cdot\cdot,x_2(k)) \\ &\vdots = \vdots \\ x_m &= (x_m(1),x_m(2),\cdot,\cdot\cdot,x_m(k)) \end{aligned} \qquad \begin{aligned} i &= 0,1,2,\cdots,m, \\ k &= 1,2,3,\cdots,n \in N \end{aligned} \tag{2}$$

If we take $x_0(k)$ as the referenced sequence, and other sequences as inspected sequences, then, it is called localization grey relational grade. If each sequence $x_i(k)$ can be the referenced sequence, then, it is called globalization grey relational grade. This research focused on five kinds of globalization grey relational grade, and the five equations (Equation 3 to Equation 7) are provided below [Wen, 21].

2.3. The Kinds of Grey Relational Grade

2.3.1. Wu's Method

$$\Gamma_{ij} = \Gamma(x_i,x_j) = \frac{\Delta_{\min.} + \Delta_{\max.}}{\overline{\Delta}_{ij} + \Delta_{\max.}},\ \overline{\Delta}_{ij} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[\Delta_{ij}(k)\right]^2} \tag{3}$$

2.3.2. Wen's Method

$$\Gamma_{ij} = \Gamma(x_i,x_j) = \frac{\Delta_{\min.} + \Delta_{\max.}}{\overline{\Delta}_{ij} + \Delta_{\max.}},\ \overline{\Delta}_{ij} = \left\{\frac{1}{n}\sum_{k=1}^{n}[\Delta_{ij}(k)]\right\} \tag{4}$$

2.3.3. Hsia's Method

$$\Gamma_{ij} = \Gamma(x_i,x_j) = \frac{\Delta_{\max.} - \overline{\Delta}_{ij}}{\Delta_{\max.} - \Delta_{\min.}},\ \overline{\Delta}_{ij} = \left\{\frac{1}{n}\sum_{k=1}^{n}\Delta_{ij}(k)\right\} \tag{5}$$

2.3.4. Nagai's Method

$$\Gamma_{ij} = \Gamma(x_i,x_j) = 1 - \frac{\overline{\Delta}_{ij}}{\Delta_{\max.}},\ \overline{\Delta}_{ij} = \left(\sum_{k=1}^{n}\left[\Delta_{ij}(k)\right]^2\right)^{\frac{1}{2}} \tag{6}$$

2.3.5. Liu's Method

$$\Gamma_{ij} = \frac{1}{1 + \left|\sum_{k=2}^{n-1}[\Delta_{ij}(k)] + \frac{1}{2}\times\Delta_{ij}(n)\right|} \tag{7}$$

where：$i = 1,2,3,\ldots,m$, $k = 1,2,3,\ldots,n$, $j \in I$

A. x_i Referenced sequence. x_j Inspected sequences

B. $\Delta_{ij} = \left|x_i(k) - x_j(k)\right|$: The difference among x_i and x_j (norm)

C. $\Delta_{\min.} = \underset{j\in i}{\overset{\min.\min.}{\forall}} \forall k \left|x_i(k) - x_j(k)\right|$, $\Delta_{\max.} = \underset{j\in i}{\overset{\max.\max.}{\forall}} \forall k \left|x_i(k) - x_j(k)\right|$.

2.4. The Calculation of Globalization Grey Relational Grade

Once the results are found, we can use the eigenvector method to rank the sequence, and then choose the optimal one. The calculation steps of globalization grey relational grade are shown below:

1. Constructing the relative weighting matrix $[R]_{m \times m}$, which is called grey relational matrix.

$$R_{m \times m} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1m} \\ \Gamma_{21} & \Gamma_{22} & \cdots & \Gamma_{2m} \\ \vdots & \vdots & \ddots & \Gamma_{11} \\ \Gamma_{m1} & \Gamma_{m2} & \cdots & \Gamma_{mm} \end{bmatrix} \quad (8)$$

2. Finding the eigenvalue for the relative weighting matrix $AR = \lambda R$
3. Using eigenvector method to find the weighting for each target $P^{-1}RP = diag\{\lambda_1, \lambda_2, \lambda_{3,.....}\lambda_n\}$
4. The maximum eigenvalue ($\lambda_{\max}$) and the maximum corresponding eigenvector are the weighting value for whole sequences.

III. Real Case

3.1. The Five Brands Automobile in Germany

There are five famous automobile brands In Germany, including VW, BMW, Benz, Audi and Porsche. Figure 1 to Figure 5 respectively show them [Shen et al., 22].

Figure 1: VW Phaeton

Figure 2: BMW 7 Series

Figure 3: Mercedes Benz S Class

Figure 4: Audi A8

Figure 5: Porsche Panamera

According to the characteristics of automobiles, the paper presents eight factors, which are shown in Table 1.

Table 1: The Germany Luxury Limousine/ Sedan Car-Actual Models 2015

Brand/ Factor	F1	F2	F3	F4	F5	F6	F7	F8
B1	3.6	280	370	17.0	90.500	8.6	2154	11.4
B2	2.9	320	450	10.2	88.700	5.7	1920	7.9
B3	3.498	306	370	10.7	87.160	6.8	1925	6.8
B4	3.993	435	600	10.1	103.00	4.5	1995	9.2
B5	3.605	310	400	10.5	83.77	6.3	1845	8.4

B1: VW Phaeton (V6, Automatic-Tiptronic, 4 Seats); B2: BMW 7 Series (740 Li);
B3: Mercedes Benz S class (S400 Hybrid); B4: Audi A8 (4.0 TFSI quattro); B5: Porsche Panamera
F1: Liter; F2;Horse power; F3: Maximum Torque [N-m]; F4 Compress ratio; F5: Price [1,000 EUR]:
F6: Acceleration time(0-100 *k*m/h) in[second]; F7: Weight; F8: Fuel consumption -combined [l/100 km]

3.2. *The Analysis Steps*

Based on the data in Table 1, the five symbols of factor x_1, x_2, x_3, x_4 and x_5 respectively stand for VW; BMW 7 series; Mercedes Benz; Audi and Porsche, which constitute the original data.

x_1=(3.6, 280, 370, 17.0, 90.500, 8.6, 2154, 11.4),
x_2=(2.9, 320, 450, 10.2, 88.700, 5.7, 1920, 7.9),
x_3=(3.498, 306, 370, 10.7, 87.160, 6.8, 1925, 6.8),
x_4=(3.993, 435, 600, 10.1, 103.900, 4.5, 1995, 9.2),
x_5=(3.605, 310, 400, 10.5, 83.277, 6.3, 1845, 8.4).

Next, by referring to the steps of calculation in globalization grey relational grade, the researcher calculated the grey relational grade.

3.2.1. *Wu's Method*

By referring to Equation (3), the researcher used x_1 as the referenced sequence and $x_2 \sim x_5$ as the inspected sequences to calculate the grey relational grade. We have Γ_{11} =1.0000, Γ_{12} =0.7981, Γ_{13} =0.7450, Γ_{14} =0.7008 and Γ_{15} =0.7578. Based on the calculation steps mentioned above, the values from Γ_{21} to Γ_{55} can be obtained. Then, the grey relational matrix from the values of Γ_{ij} can be built up:

$$R_{5\times5} = \begin{bmatrix} 1.0000 & 0.7891 & 0.7450 & 0.7008 & 0.7578 \\ 0.8348 & 1.0000 & 0.7479 & 0.8501 & 0.7892 \\ 0.8438 & 0.8026 & 1.0000 & 0.7378 & 0.9387 \\ 0.8125 & 0.8860 & 0.7378 & 1.0000 & 0.7647 \\ 0.8297 & 0.8120 & 0.9280 & 0.7323 & 1.0000 \end{bmatrix}$$

The maximum eigenvalue of $R_{5\times5}$ = 4.2070, and the maximum eigenvalue corresponding eigenvector are (0.4228, 0.4483, 0.4601, 0.4460, 0.4579).

3.2.2. *Wen's Method*

Just the same as Wu's calculation method, the grey relational matrix is

$$R_{5\times5} = \begin{bmatrix} 1.0000 & 0.9809 & 0.8656 & 0.8518 & 0.8800 \\ 0.9858 & 1.0000 & 0.8854 & 0.8973 & 0.8965 \\ 0.9226 & 0.9137 & 1.0000 & 0.8489 & 0.9899 \\ 0.9141 & 0.9229 & 0.8499 & 1.0000 & 0.8564 \\ 0.9195 & 0.9091 & 0.9880 & 0.8339 & 1.0000 \end{bmatrix}$$

The maximum eigenvalue of $R_{5\times5}$ = 4.6228, and the maximum eigenvalue corresponding eigenvector are (0.4429, 0.4513, 0.4525, 0.4391, 0.4501).

3.2.3. *Hsia's Method*

Just the same as Wu's calculation method, the grey relational matrix is

$$R_{5\times5} = \begin{bmatrix} 1.0000 & 0.9805 & 0.8448 & 0.8261 & 0.8637 \\ 0.9856 & 1.0000 & 0.8706 & 0.8856 & 0.8846 \\ 0.9161 & 0.9056 & 1.0000 & 0.8221 & 0.9898 \\ 0.9060 & 0.9165 & 0.8221 & 1.0000 & 0.8323 \\ 0.9125 & 0.9000 & 0.9879 & 0.8008 & 1.0000 \end{bmatrix}$$

The maximum eigenvalue of $R_{5\times5}$ = 4.5714, and the maximum eigenvalue corresponding eigenvector are (0.4417, 0.4525, 0.4536, 0.4376, 0.4505).

3.2.4. *Nagai's Method*

Just the same as Wu's calculation method, the grey relational matrix is

$$R_{5\times5} = \begin{bmatrix} 1.0000 & 0.5935 & 0.4792 & 0.3506 & 0.5138 \\ 0.5935 & 1.0000 & 0.3077 & 0.6379 & 0.4516 \\ 0.4792 & 0.3077 & 1.0000 & 0.0000 & 0.8162 \\ 0.3506 & 0.6379 & 0.0000 & 1.0000 & 0.1340 \\ 0.5138 & 0.4516 & 0.8162 & 0.1340 & 1.0000 \end{bmatrix}$$

The maximum eigenvalue of $R_{5\times5}$ = 2.7635, and the maximum eigenvalue corresponding eigenvector are (0.4864, 0.4785, 0.4438, 0.3073, 0.4930).

3.2.5. *Liu's Method*

Just the same as Wu's calculation method, the grey relational matrix is

$$R_{5\times5} = \begin{bmatrix} 1.0000 & 0.0027 & 0.0037 & 0.0018 & 0.0026 \\ 0.0027 & 1.0000 & 0.0096 & 0.0028 & 0.0070 \\ 0.0037 & 0.0096 & 1.0000 & 0.0022 & 0.0083 \\ 0.0018 & 0.0028 & 0.0022 & 1.0000 & 0.0020 \\ 0.0026 & 0.0070 & 0.0083 & 0.0020 & 1.0000 \end{bmatrix}$$

The maximum eigenvalue of $R_{5\times5}$ = 1.0191, and the maximum eigenvalue corresponding eigenvector are (0.2763, 0.5425, 0.5705, 0.2180, 0.5064).

The cardinal of each brand and the average are shown in Table 2 and Fig. 6.

Table 2: The Weighting of Five Brands of Automobiles

Method	VW Phaeton	BMW 7 Series	Mercedes Benz *S* Class	Audi A8	Porsche Panamera
Wu	0.4228	0.4483	0.4601	0.4460	0.4579
Wen	0.4429	0.4513	0.4525	0.4391	0.4501
Hsia	0.4417	0.4525	0.4536	0.4376	0.4505
Nagai	0.4864	0.4785	0.4438	0.3073	0.4930
Liu	0.2763	0.5425	0.5705	0.2180	0.5064
Average	0.41402	0.47462	0.4761	0.3696	0.47158
Rank	4	2	1	5	3

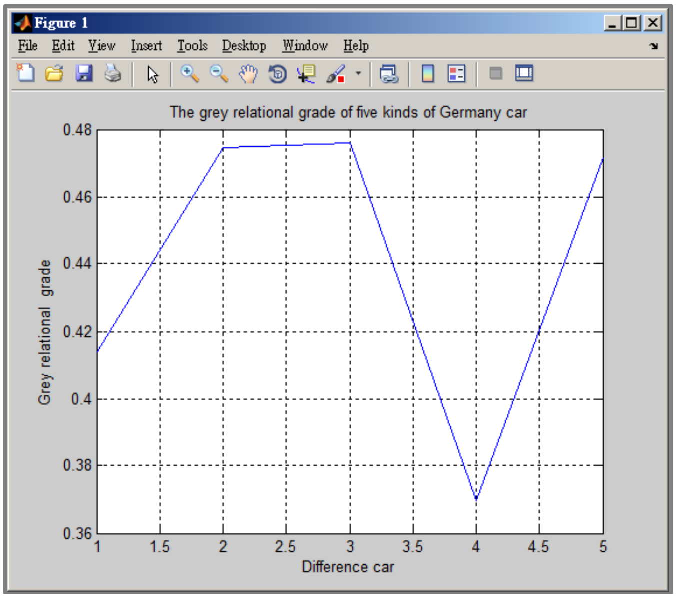

Figure 6: The Grey Relational Grade of Five Kinds of Germany's Automobiles

IV. Conclusion

Former research on automobile purchase tended to apply fuzzy MCDM and AHP. Other relevant research is found to focus on automotive management. Concerning the application of the grey system theory, only GM(0,N), MCDM and LGRA are found to be applied. Therefore, the proposed cardinal globalization grey relational grade analysis applied in the paper can be regarded as the pioneer in this field. Besides, all analysis factors are drawn from the official websites of Germany's automobile companies with credibility.

Through actual value analysis and calculation by using cardinal globalization grey relational grade, we found the ranking of the automobile manufacturers is as follows: Porsche, BMW, Benz, VW and Audi. However, the grey relational grade among the five kinds automobile, which is around 0.4412, does not manifest significant difference. It means that the five brands are almost on the same level as to quality. Consumers do not develop any inclination or sense of favor upon buying automobiles.

To sum up, the paper only used globalization grey relational grade in the soft-calculation to conduct preliminary study. If other soft computing methods, such as grey entropy or rough set theory, can be further applied to verify the data in a cross-analysis way or more influencing factors can be obtained to analyze the data, the results can be more convincing and more precise.

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Kun-Li Wen received his B. S. degree in E.E. and M. S. degree in Automation Engineering from Fengchia University, Taichung, Taiwan in 1980 and 1983 respectively. In 1997, he received the Ph. D. degree in Mechanical Engineering from National Central University, Chungli, Taiwan. He is now a Full Professor in Department of Electrical Engineering, Chienkuo Technology University and the Secretary General in Taiwan *Kansei* Information Society. His research interests are grey system analysis, rough set theory and *kansei* engineering.100-200 words. Biography should identify where the author earned his/her highest degree, present affiliation and position, current research interests, and number of papers published, number of conferences/seminars attended. After first sentence you can use pronoun of the person (he or she) and not the author’s first name and last name. List any memberships in professional societies separated by commas. Personal hobbies will be deleted from the biography.

Keng-Hao Chang was born on 1973; he is now a graduate student of the Department of Electrical Engineering, Chienkuo Technology University, Changhua. Currently he is in the National Chung-Shan Institute of Science & Technology, his research interests in the field of system engineering and grey system analysis.

Yi-Cheng Shen meantime is a junior student in Department of Electrical Engineering, Chienkuo Technology University. His major interest is in environment analysis and application of material in electrical engineering.

