A Study of Mixed Convection in an Enclosure with Different Inlet and Outlet Configurations

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***Abstract*—**A constant flux heat source was heated vertical wall with the fluid considered being air. The other side walls including the top and bottom of the enclosure were assumed to be adiabatic. The inlet opening, located on the left vertical wall, was placed at varying locations. The outlet opening was placed on the opposite heated wall at a fixed location. The basis of the investigation was the two–dimensional numerical solutions of governing equations by using Finite Difference Method (FDM).Significant parameters considered were Richardson number (*Ri*) and Reynolds number (*Re*). Results are presented for Richardson number 0 to 10 at *Pr*=0.71 and *Re*=50,100,200.The effects of Richardson number and position of the inlet on dimensional temperature inside the enclosure was investigated. The resulting interaction between forced external air stream and buoyancy-driven flow by the heat source are presented in the form of velocity profile and temperature distribution within the enclosure by patterns of graphs. The computational results indicate that heat transfer is strongly affected by Reynolds and Richardson numbers. As the value of *Ri* increases, there occurs a transition from forced convection to buoyancy dominated flow at *Ri*>1. A detailed analysis of flow pattern shows that natural or forced convection is based on the parameter *Ri*.

***Keywords*—**Crank Nicolson Numerical Scheme; Finite Difference Method; Reynolds Number; Richardson Number; The Heated Vertical Wall; The Partial Differential Equations.

***Abbreviations*—**Convective Boundary Conditions (CBC); Finite Difference Method (FDM); Forward Difference Equation (FDE); Forward Difference Scheme (FDS); Partial Differential Equations (PDEs); Reynolds Number (Re); Richardson Number (Ri).

I. Introduction and Literature Review

1.1. Background of Study

THERMAL buoyancy forces play a significant role in forced convection heat transfer when the flow velocity is relatively small and the temperature difference between the surface and the free stream is relatively large. The buoyancy forces modify the flow and temperature fields and hence the heat transfer rate from the surface. Problems of heat transfer in enclosures by mixed convection has been the subject of investigations for many years. Numerous experimental and numerical studies of mixed convection in a cavity have been conducted by a great number of researches. Mixed convection occurs in many heat transfer devices such as a the cooling system of a nuclear power plant, large heat exchangers, cooling of the electronic equipment, ventilation in buildings and solar collectors. The relative direction between the buoyancy force and the externally forced flow is important. In the case where the fluid is externally forced to flow in the same direction as the buoyancy force the mode of heat transfer is termed as assisting mixed convection. In the case where the fluid externally forced to flow in the opposite direction to the buoyancy force the mode of heat transfer is termed as opposing mixed convection. Different heating conditions of the cavity as well as ventilation systems can induce different kinds of heated buoyancy flows which enhance the heat transfer in different ways. For opposing mixed convection in a cavity, when the buoyancy parameter is not large the small amount of buoyant flow induced along the heated wall can either aid or resist the main flow and cause either enhancement or reduction in the heat transfer. When the buoyancy parameter becomes large, the resulting heated buoyant flow along the side wall becomes substantial. Depending upon the flow position of the main stream, the buoyant flow can cause different kinds of flow reversals which will alter the entire flow characteristics and enhance the heat transfer in different ways. For vertical enclosures, heating is usually from the side. Therefore, enhancement in the heat transfer can be obtained from the initial position of the main flow. This main flow is usually a cold forced flow which forms mushroom-shaped plumes associated with vortices. The transition into different flows depends on the magnitudes of the Reynolds number and Richardson number. It is evident that mixed convection in a cavity with different placement of ventilation system differs so drastically that studies on the flow and heat transfer characteristics must be carefully studied.

1.2. Definition of Terms

Heat transfer: This is the process through which heat moves from one body or substance to another by conduction, radiation, convection or a combination of any of these modes.

Mass transfer: This is the mass in transit which arise as result of concentration difference in a mixture. It may include bulk mass transfer from convection process.

Laminar flow: This is the flow of the fluid in which adjoining layers of fluid flow parallel to one another. All the fluids particles move in distinct and separate layer without mixing within layers.

Turbulent flow: This is the flow of fluid in which its velocity at any point varies rapidly in an irregular manner.

Viscosity: This is an internal property of fluid that offers resistance to flow.

Thermal conductivity: This is the quantity of heat transmitted through unit thickness in direction normal to a surface.

Specific heat: This is amount of heat per unit mass required to raise the temperature by one Kelvin.

1.3. Literature Review

Various researchers have carried out investigations into the effect of mixed convective flows in rectangular enclosures by using analytical, experimental, and numerical methods. Angirasa [1] presented a numerical study of mixed convection of airflow in an enclosure with an isothermal vertical wall. Forced conditions were imposed by providing an inlet and a vent in the enclosure. Both positive and negative temperatures potential were considered by varying the Grashof number from -10^6 to 10^6. In their study, steady – state solutions could not be obtained for higher positive values of the Grashof number and for buoyancy- dominated flows. In general forced flows help to enhance heat transfer for both negative and positive values of Grashof numbers. Later a numerical analysis of laminar mixed convection in an open cavity with a heated wall bounded by a horizontally insulated plate was presented by Manca et al., [7]. Three heating modes were considered: assisting flow, opposing flow and heating from below. Results for Richardson numbers equal to 0.1 and 100, Re= 100 and 1000 and aspect ratio in the range 0.1 -1.5 were reported. It was shown that that maximum temperature values were decreased as the Reynolds and the Richardson number increased. The effect of the ratio of channel height to the cavity height was found to play a significant role on streamline and isotherm patterns for different heating configuration. The investigation showed that opposing forced flow configurations had the highest thermal performance in terms of both maximum temperature and average Nusselt number. Later, similar problems for the case of the assisting forced flow configuration were tested experimentally by Manca et al., [8] and based on the flow visualization results, they pointed out that for Re=1000, there are two nearly distinct fluid motions: a parallel forced flow in the channel and a recirculation flow inside the cavity. For Re=100, the effect of a stronger buoyancy force determined the penetration of thermal plume from the heated plate wall into the upper channel. The stability of mixed-convective flows has been analysed by Leong et al.,[5] for an open cavity heated from the bottom wall. Their analysis concludes that transition to the mixed convection regime depends on the relative magnitude of the Grashof and Reynolds numbers of the flow. A three-dimensional study of mixed convection cooling of multiple heat source flush- mounted on the bottom surface of a horizontal rectangular duct was performed by Wang & Jaluria (2002), Yamada & Ichimiya [15] and later validation of mixed convection in a differentially heated air-cooled cavity was done by several researchers Moraga & Lopez [9], Lo et al., [6] and Benzaoui [2]. The effect of the exit port locations and the aspect ratio of the heat generating body on the heat transfer characteristics, as well as the entropy generation in a square cavity were investigated by Shuja et al., [11]. They found that the overall normalized Nusselt number as well as irreversibility was strongly affected by both the location of the exit port and the aspect ratio. Singh & Sharif [13] studied mixed convection in an air-cooled rectangular cavity with differentially heated vertical isothermal side walls having inlet and exit port. Several different placement configurations of the inlet and exit ports were investigated. The best configuration was selected by analyzing the cooling effectiveness of the cavity which suggested that injecting air through the bottom of cold wall and exiting near the top of the hot wall was more effective in heat removal. The forced and natural convection assist each other in the heat removal process. Studies were also undertaken by Sigey et al., [12] who studied buoyancy driven free convection turbulent heat transfer in an enclosure. They investigated a three- dimensional enclosure containing a convectional heater built into one wall having a window in same wall. The heater is located below the window and the other remaining wall insulated. The result showed there were three regions: a cold upper region, a hot region in the area in between and a warm lower region. Hamid Reza Goshayeshi & Mohammad Reza Safaiy [4] carried out an investigation of turbulent mixed convection in air- filled enclosure. The result showed that when Reynolds number increases the circulation of flow vortices increases and becomes stronger, making the forced convection effective and more dominant for different values of Richardson number. Also in large Richardson numbers, the natural convection is a major parameter of heat transfer in a cavity. Chu & Yun [3] studied flow behavior and heat transfer in an inclined rectangular enclosure subjected to a moving lid and temperature differential. The numerical results show that there are three kinds of flow regime in a rectangular cavity inclined from 0 to 360: buoyancy dominant, inertia dominant and intermediate transition (mixed convection flow).The combination Re=100, Gr=1000 and $\theta= 0^0$ induces excellent thermal performance corresponding to wavy profiles in local Nusselt number. The study also revealed that good thermal performance within a local region can generate a higher friction force on the neighboring boundary. Now in this study, numerical simulations are carried out over a range of Richardson numbers and Reynolds numbers to measure and quantify the best possible inlet positions to obtain minimum temperature inside the enclosure. Also the temperature and the velocity profiles in the mid-sections of the cavity are presented. The dependence of the thermal and flow fields on the location of the inlet opening are studied in detail. The researcher has applied finite difference method to carry out the numerical simulations to investigate laminar mixed convection cooling in a rectangular enclosure. In the present flow configuration, the heated wall is placed on the outflow side that provides the highest thermal performance as compared with heated wall placed at the top or at the bottom [Manca et al., 7]. Incoming flow is at the ambient temperature, T_i, and the outgoing flow will be assumed to have zero diffusion flux for all variables which are known as convective boundary conditions (CBC) [Sani & Gresho, 10; Sohankar et al., 14]. All solid boundaries will be assumed to be rigid with no-slip. The effect of the placement of the inlet on the thermal performance is taken into special consideration. Based on the survey, it was found that no work has been reported for mixed convectional in a vented rectangular enclosure with varying inlet port location on one vertical wall and outlet port location fixed at the top of the opposite heated vertical wall.

1.4. Statement of the Problem

The problem of air circulation in buildings or other enclosures where heat is generated is of major concern to engineers and designers because of its almost universal occurrence in many applications. No work has been done on mixed convectional in a vented rectangular enclosure with varying inlet port location on one vertical wall and outlet port location fixed at the top of the opposite heated vertical wall. Therefore the researcher has the desire to use finite difference method (numerical techniques) to provide for deeper understanding of the flow and heat transfer mechanism in a rectangular enclosure with uniformly heated vertical wall with outlet placed opposite the inlet opening where air enters at some speed.

1.5. Justification of the Study

This study will be very important for engineers and designers constructing air-cooled enclosures such as buildings, factories and other structures where rapid cooling is required. This study once complete, will provide a numerical method of obtaining data that could be used by engineers and designers in the construction of enclosures with efficient cooling systems.

1.6. General Objectives

To establish using finite difference method (numerical technique) the velocity profile and temperature distribution in an enclosure by changing the position of the inlet opening for a range of Richardson number and Reynolds number.

1.7. Specific Objectives

i) To establish the effects of Richardson number and Reynolds number on temperature distribution within the enclosure.
ii) To establish the effects of Richardson number and Reynolds number on the velocity of flow inside the enclosure.
iii) To establish the best location of the inlet that obtains minimum temperature inside the enclosure.

1.8. Geometry of the Problem

The details of the geometry for the inlet locations considered are shown in figure 1

A Cartesian coordinate system is used with the origin at the lower left hand corner of the computational domain. The model considered here is a rectangular enclosure with uniform constant- flux heat source, applied on the right vertical wall. The enclosure dimensions are defined by height H and width L. The other side walls including top and bottom of the enclosure are assumed to be adiabatic. The inflow opening, located on the left vertical wall, is arranged as shown in figure 1 and may vary location at a distance h_i from the bottom of the enclosure. The outflow opening of the cavity is fixed at the top of the opposite heated wall and the size of the inlet port is the same as the outlet port. The inlet port location is altered along the left wall. In each simulation only one inlet port location is considered. Consequently after simulation only the inlet port location is moved away from its initial location and the simulation is repeated for the new location of the inlet port. It is assumed that the incoming flow is at a uniform velocity, u_i and at the ambient temperature. The details of the geometry for the configurations are shown in the figure 1 below.

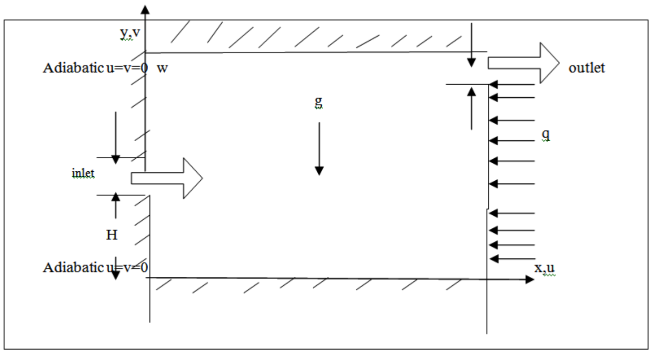

Figure 1: Schematic Diagram of the Problem considered and Coordinate

II. Mathematical Formulation

2.1. Governing Equations

To model the flow under study, we have used the conservation equations for mass, momentum and energy for a two-dimensional steady, laminar flow. For the moderate temperature difference to be considered in this work, all the physical properties of the fluid μ, κ, ρ and Cp will be considered constant except density in the buoyancy term, which obeys Boussinesq approximation. In the energy conservation equation we shall neglect the effects of compressibility and viscous dissipation. Thus the general equations that govern the flow can be written as:

2.2. Continuity Equation

The continuity equation is essentially the equation for conservation of mass that is matter may neither be created nor destroyed. In Cartesian coordinates, it is convenient to consider a two- dimensional flow, assume the velocity components to be u and υ in the x and y directions respectively. When the density, ρ, of the fluid is treated as constant, we have the continuity equation given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

2.3. Momentum Equation

The momentum equations are derived from Newton's second law of motion. This law requires that the sum of all forces acting on the fluid must be equal to the rate of increase of the fluid momentum. The external forces acting on a fluid particle are of two types: body forces which are proportional to volume and which act on the fluid particle from an external force field such as gravitational, electric, magnetic fields; and surface forces which are proportional to area and which result from the stresses such as static pressure and viscous stresses acting on the surface.

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \eta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta\left(T - T_O\right) \tag{3}$$

Equation (2) and (3) are the Navier-Stokes equations for steady two-dimensional flow of an incompressible, constant property fluid.

2.4. Energy Equation

The energy equation is derived by applying the first law of thermodynamics. For a flow field without any heat sources and neglecting radiation, energy balance about a point this equation can be written as: $\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$

Where dQ is the heat added to the particle in time dt. This amount of heat will increase the internal energy of the particle by dE while performing an amount of work dW. For an incompressible, steady flow, the energy equation is written as:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

With the boundary conditions

$u = u_i$, $v = 0$ and $T = T_i$ at the inlet

$v = 0$, $p = 0$, $\frac{\partial T}{\partial x} = 0$ at the outlet

$u = 0$, $v = 0$, $q = \kappa\frac{\partial T}{\partial x}$ along the heated wall

$u = v = 0$, $\frac{\partial T}{\partial x} = 0$ along the vertical insulated wall,

$u = v = 0$, $\frac{\partial T}{\partial y} = 0$ along the horizontal insulated walls.

where x and y are the distances measured along the horizontal and vertical directions respectively; u and v are the velocity components in the x- and y-direction respectively; T denotes the temperature; η and α are the kinematics viscosity and the thermal diffusivity respectively; p is the pressure and q is the uniform constant heat flux. ρ is the density of the fluid.

A dimensionless form of governing equations can be obtained by introducing the following dimensionless variables: $X = \frac{x}{L}$, $Y = \frac{y}{L}$, $u = \frac{u}{u_i}$, $V = \frac{v}{u_i}$, $P = \frac{P}{\rho u_i^2}$ $\theta = \frac{T-Ti}{Th-Ti} = \frac{T-Ti}{qL/k}$

Based on the dimensionless variables above, the governing equations (1) to (4) reduces to non-dimensional form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{5}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{\mathrm{Re}^2}\theta \tag{7}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{8}$$

Where $G^r = \frac{g\beta q H^4}{\kappa\eta^2}$, $Re = \frac{uH}{\eta}$, $Pr = \frac{\eta}{\alpha} = \frac{\eta C_p}{\kappa}$, $Ri = \frac{Gr}{Re^2}$. The dimensionless form of the boundary condition is

U=1, V=0, $\theta = 0$ at the inlet:

P=0, at the outlet (CBC)

U=0, V=0, $\frac{\partial \theta}{\partial \mathrm{N}}$=0, at the cavity walls (except the right vertical wall)

U=0, V=0, $\frac{\partial \theta}{\partial X}$ = -1, along the heated right vertical wall.

U=0, V=0, $\frac{\partial \theta}{\partial Y}$ =0, $\frac{\partial \theta}{\partial X} = 0$, along the vertical insulated wall.

Here X and Y are dimensionless coordinates varying along horizontal and vertical axes respectively: U and V are dimensionless velocity components in X- and Y- directions respectively; $\theta$is the dimensionless temperature; P is the dimensionless pressure and N is the non-dimensional distance in either x or y direction acting normal to the surface.

The Reynolds number is based on the inlet velocity and the enclosure length and indicates the ratio of inertial forces to viscous forces in a fluid, whereas the Grashof number is based on the constant heat flux applied at the heated wall and indicates the ratio of buoyancy forces in a fluid to viscous forces. In the above system of equations, all distances are normalized by L, velocities are normalized by inlet velocityu_i and pressure normalized byρu_i^2 where ρ is the density of the fluid.

The temperature is normalized by $\theta = \frac{T-T_i}{\frac{qL}{k}}$. The Richardson number, defined as Ri = $\frac{Gr}{Re^2}$ is a characteristic number for the mixed convection process that indicates the ratio of natural convection to forced convection or relative dominance of the natural overforced convection effects.

III. Methods of Solution

3.1. Introduction

In this section the method and procedure of solving the problem is discussed.

3.2. Computational Procedure

In this study A Hybrid scheme is developed and finite difference method is used to solve the momentum and energy equations. The method obtains a finite system of linear or nonlinear algebraic equations from momentum and energy. The Partial Differential Equations are solved by discretizing the given PDE and coming up with the numerical schemes analogue to the equations subject to the given boundary conditions. MATLAB software is used to generate solution values in this study.

3.3. Governing Equations

The viscous incompressible flow and the temperature distribution inside the cavity are described by the momentum and energy equations. Systems of Navier-Stokes and energy partial differential equations with appropriate boundary conditions governing our problem are solved using Finite Difference Method. The fluid flow is expressed theoretically by momentum and energy questions under the assumption that the fluid flow is steady, lamina, incompressible and two-dimensional.

3.4. Discretization of the Governing Equations

Considering momentum and energy equations;

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{9}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta \tag{10}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{11}$$

In the current investigation, the Richardson number is defined as;

$$Ri = \frac{Gr}{Re^2}$$

Where Grashof number, Reynolds number and Prandtl number are defined as;

$$Gr = \frac{g\beta q L^4}{\eta^2 \kappa}, Re = \frac{u_i L}{\eta}, Pr = \frac{\eta}{\alpha} = \frac{\mu C_p}{\kappa}$$

The Reynolds number is based on the inlet velocity and the enclosure length whereas the Richardson number is based on the constant heat flux applied at the heated wall.

3.5. Horizontal Velocity

Hybrid scheme,$U_x$is replaced by forward difference approximation while U_{xx} and U_{yy} is replaced by central difference approximation, equation (9)becomes

$$\left[\frac{U_{i+1,j} - U_{i,j}}{\Delta x}\right] = -3.6\times 10^{-4} + \frac{1}{200}\left(\left[\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta x)^2}\right] + \left[\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta y)^2}\right]\right)$$

We investigate the effect of *Re*, *Ri* on the fluid velocity. Taking,$\Delta x = \Delta y = 0.1$, *Re*=50, 100,200, *Ri*=0, 5, 10, $P = 3.6 \times 10^{-4}$ with boundary conditions V=0, U=1, $\theta =$ 0we get the scheme**.**

$$26.5U_{i+1,j} - 23.5U_{i,j} - U_{i-1,j} = U_{i,j-1} + U_{i,j+1} - 2 \tag{12}$$

Taking i=1,2,3………..10andj=1we form the following systems of linear algebraic equations

$$26.5U_{2,1} - 23.5U_{1,1} - U_{0,1} = U_{1,0} + U_{1,2} - 2$$
$$26.5U_{3,1} - 23.5U_{2,1} - U_{1,1} = U_{2,0} + U_{2,2} - 2$$
$$26.5U_{4,1} - 23.5U_{3,1} - U_{2,1} = U_{3,0} + U_{3,2} - 2$$
$$26.5U_{5,1} - 23.5U_{4,1} - U_{3,1} = U_{4,0} + U_{4,2} - 2$$
$$26.5U_{6,1} - 23.5U_{5,1} - U_{4,1} = U_{5,0} + U_{5,2} - 2$$
$$26.5U_{7,1} - 23.5U_{6,1} - U_{5,1} = U_{6,0} + U_{6,2} - 2$$
$$26.5U_{8,1} - 23.5U_{7,1} - U_{6,1} = U_{7,0} + U_{7,2} - 2$$
$$26.5U_{9,1} - 23.5U_{8,1} - U_{7,1} = U_{8,0} + U_{8,2} - 2$$
$$26.5U_{10,1} - 23.5U_{9,1} - U_{8,1} = U_{9,0} + U_{9,2} - 2$$
$$26.5U_{11,1} - 23.5U_{10,1} - U_{9,1} = U_{10,0} + U_{10,2} - 2$$

The above algebraic equations can be written in matrix form as when U(x,0)=0and U(x,y)=sin x+siny

$$\begin{bmatrix} -23.5 & 26.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -23.5 & 26.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -23.5 & 26.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -23.5 & 26.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -23.5 & 26.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -23.5 & 26.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -23.5 & 26.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -23.5 & 26.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -23.5 & 26.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -23.5 \end{bmatrix} \begin{bmatrix} U_{11} \\ U_{21} \\ U_{31} \\ U_{41} \\ U_{51} \\ U_{61} \\ U_{71} \\ U_{81} \\ U_{91} \\ U_{101} \end{bmatrix} = \begin{bmatrix} -1.048796 \\ -1.103435 \\ -1.15774572 \\ -1.2115635 \\ -1.26472736 \\ -1.31707746 \\ -1.36845646 \\ -1.4187099 \\ -1.4676869 \\ -1.51523999 \end{bmatrix} \tag{13}$$

Solving the above matrix equation, we get the solutions for changing *Re*.

3.6. Vertical Velocity

Discretizing the vertical velocity equation (10) becomes

$$\left[\frac{V_{i+1,j}-V_{i,j}}{\Delta y}\right]=\frac{1}{50}\left(\left[\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{(\Delta x)^2}\right]+\left[\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{(\Delta y)^2}\right]\right)+2\left(\frac{\theta_{i,j+1}+\theta_{i,j}}{2}\right) \quad (14)$$

We investigate the effect of *Re*, *Ri* on the fluid velocity. Taking, $\Delta x = \Delta y = 0.1$, *Re*=100, 10, 1 *Ri*=2, 5, 10 and we get the scheme

$$V_{i+1,j}-4V_{i,j}+V_{i-1,j}=-V_{i,j+1}+V_{i,j-1}-1 \quad (15)$$

Taking i=1,2,3…….10 and j=1, we form the following systems of linear algebraic equations (i is taken to be y and j is taken to be x)

$$-4V_{1,1}+V_{2,1}+V_{0,1}=-V_{1,2}+V_{1,0}-1$$
$$-4V_{2,1}+V_{3,1}+V_{1,1}=-V_{2,2}+V_{2,0}-1$$
$$-4V_{3,1}+V_{4,1}+V_{2,1}=-V_{3,2}+V_{3,0}-1$$
$$-4V_{4,1}+V_{5,1}V+_{3,1}=-V_{4,2}+V_{4,0}-1$$
$$-4V_{5,1}+V_{6,1}+V_{4,1}=-V_{5,2}+V_{5,0}-1$$
$$-4V_{6,1}+V_{7,1}V+_{5,1}=-V_{6,2}+V_{6,0}-1$$
$$-4V_{7,1}+V_{8,1}+V_{6,1}=-V_{7,2}+V_{7,0}-1$$
$$-4V_{8,1}+V_{9,1}+V_{7,1}=-V_{8,2}+V_{8,0}-1$$
$$-4V_{9,1}+V_{10,1}+V_{8,1}=-V_{9,2}+V_{9,0}-1$$
$$-4V_{10,1}+V_{11,1}+V_{9,1}=-V_{10,2}+V_{10,0}-1$$

The above algebraic equations can be written in matrix form when initial and boundary conditions $U_{1,j+1}$=1and$U_{1,j}$=1

$$\begin{bmatrix}-4&1&0&0&0&0&0&0&0&0\\1&-4&1&0&0&0&0&0&0&0\\0&1&-4&1&0&0&0&0&0&0\\0&0&1&-4&1&0&0&0&0&0\\0&0&0&1&-4&1&0&0&0&0\\0&0&0&0&1&-4&1&0&0&0\\0&0&0&0&0&1&-4&1&0&0\\0&0&0&0&0&0&1&-4&1&0\\0&0&0&0&0&0&0&1&-4&1\\0&0&0&0&0&0&0&0&1&4\end{bmatrix}\begin{bmatrix}V_{11}\\V_{21}\\V_{31}\\V_{41}\\V_{51}\\V_{61}\\V_{71}\\V_{81}\\V_{91}\\V_{101}\end{bmatrix}=\begin{bmatrix}-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\end{bmatrix} \quad (16)$$

Solving the above matrix equation, we get the solutions for changing *Ri*.

3.7. Temperature

Discretizing the temperature equation (22) becomes

$$\frac{\theta_{i+1,j}-\theta_{i,j}}{(\Delta x)}+\frac{\theta_{i,j+1}-\theta_{i,j}}{(\Delta y)}=\frac{1}{200\times 0.71}\left[\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta x)^2}+\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}\right] \quad (17)$$

We investigate the effect of Re, at Pr=0.71on the fluid velocity. Taking $\Delta x = \Delta t = 0.1$, Re=200, 100, 10 and U=1, V=1, we get the scheme

$$2.9\theta_{i,j}-0.45\theta_{i+1,j}-\theta_{i-1,j}=1.55\theta_{i,j+1}+\theta_{i,j-1} \quad (18)$$

Taking i=1,2,3…….10 and j=1, we form the following systems of linear algebraic equations

$$2.9\theta_{1,1}-0.45\theta_{2,1}-\theta_{0,1}=1.55\theta_{1,2}+\theta_{1,0}$$
$$2.9\theta_{2,1}-0.45\theta_{3,1}-\theta_{1,1}=1.55\theta_{2,2}+\theta_{2,0}$$
$$2.9\theta_{3,1}-0.45\theta_{4,1}-\theta_{2,1}=1.55\theta_{3,2}+\theta_{3,0}$$
$$2.9\theta_{4,1}-0.45\theta_{5,1}-\theta_{3,1}=1.55\theta_{4,2}+\theta_{4,0}$$
$$2.9\theta_{5,1}-0.45\theta_{6,1}-\theta_{4,1}=1.55\theta_{5,2}+\theta_{5,0}$$
$$2.9\theta_{6,1}-0.45\theta_{7,1}-\theta_{5,1}=1.55\theta_{6,2}+\theta_{6,0}$$
$$2.9\theta_{7,1}-0.45\theta_{8,1}-\theta_{6,1}=1.55\theta_{7,2}+\theta_{7,0}$$
$$2.9\theta_{8,1}-0.45\theta_{9,1}-\theta_{7,1}=1.55\theta_{8,2}+\theta_{8,0}$$
$$2.9\theta_{9,1}-0.45\theta_{10,1}-\theta_{8,1}=1.55\theta_{9,2}+\theta_{9,0}$$
$$2.9\theta_{10,1}-0.45\theta_{11,1}-\theta_{9,1}=1.55\theta_{10,2}+\theta_{10,0}$$

The above algebraic equations are written in matrix form when initial and boundary conditions

$$\theta(x,t)=25,\ \theta(x,0)=20$$

$$\begin{bmatrix}10&1&0&0&0&0&0&0&0&0\\1&10&1&0&0&0&0&0&0&0\\0&1&10&1&0&0&0&0&0&0\\0&0&1&10&1&0&0&0&0&0\\0&0&0&1&10&1&0&0&0&0\\0&0&0&0&1&10&1&0&0&0\\0&0&0&0&0&1&10&1&0&0\\0&0&0&0&0&0&1&10&1&0\\0&0&0&0&0&0&0&1&10&1\\0&0&0&0&0&0&0&0&1&10\end{bmatrix}\begin{bmatrix}\theta_{11}\\\theta_{21}\\\theta_{31}\\\theta_{41}\\\theta_{51}\\\theta_{61}\\\theta_{71}\\\theta_{81}\\\theta_{91}\\\theta_{101}\end{bmatrix}=\begin{bmatrix}83.75\\63.75\\63.75\\63.75\\63.75\\63.75\\63.75\\63.75\\63.75\\63.75\end{bmatrix} \quad (19)$$

Solving the above matrix equation, we get the solutions for changing *Re*.

IV. Results and Discussion

4.1. Effects of Reynolds and Richardson Numbers on Horizontal Velocity Profiles

Table 1: Horizontal Velocity for Varying Reynolds Number

Height (h_i)	*Re*=200	*Re*=100	*Re*=50
0	0.3445064	0.3245064	0.3083727
1	0.27264229	0.25264229	0.219306
2	0.2255601	0.2055601	0.1646759
3	0.1836313	0.1636313	0.1201513
4	0.1472242	0.1272242	0.08506548
5	0.11583611	0.09583611	0.05781234
6	0.08903583	0.06903583	0.0370924
7	0.0664185	0.0464185	0.02180758
8	0.04760562	0.02760562	0.0110316
9	0.03224264	0.01224264	0.00398201

The results in the table 1 above is represented graphically as seen in figure 2 below

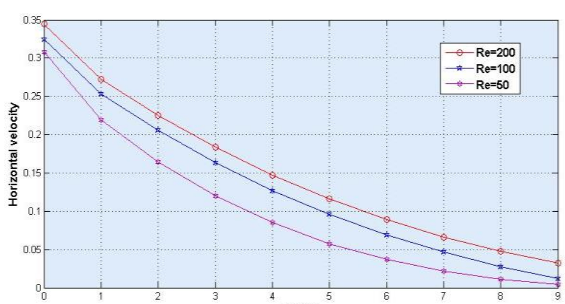

Figure 2: Horizontal Velocity Graph against Enclosure Height when Varying the ReNumber

As the height of the inlet port is increased, the velocity decreases for a particular value of Re. When the Re is large the viscous damping action becomes comparatively less and the fluid velocity increases. But as Re decreases viscous forces dominate and this retard fluid motion as shown in Table 1 and the graph on figure 2. The viscous forces tend to resist fluid motion as this viscosity decreases with increase in temperature.

4.2. Effects of Reynolds and Richardson numbers on Vertical Velocity Profiles

Table 2: Vertical Velocity for Varying Reynolds Number at $Ri = 2$

Height (h_i)	Re=50	Re=100	Re=200
0	0.3911858	0.1955929	0.0977965
1	0.5159474	0.2579737	0.1289869
2	0.569168	0.284584	0.142292
3	0.6029788	0.3014894	0.1507447
4	0.6311839	0.3155919	0.15779595
5	0.6570296	0.3285148	0.1642574
6	0.6798571	0.3399286	0.1699643
7	0.69339424	0.3469712	0.1734856
8	0.6772027	0.3386014	0.1693007
9	0.5971817	0.2735908	0.1367954

The results in the table 2 above is represented graphically as seen in figure 3 below

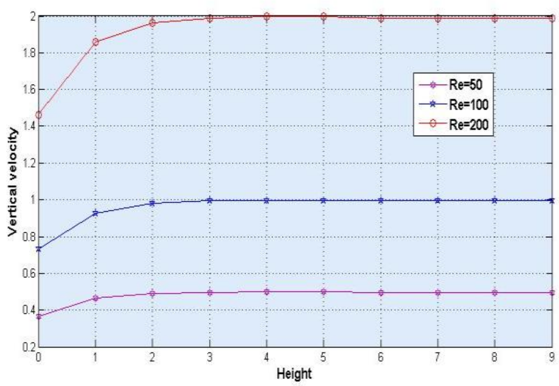

Figure 3: Vertical Velocity Graph against Enclosure Height when Varying the ReNumber

As the height of the inlet above the base increases, the vertical velocity also increases for a fixed value of Re up to a height of 2 when it remains constant. This is due to the fact that when the inlet is at the base fluid entering the enclosure mixes with cold air at the base of the vertical wall. As the height increases the incoming fluid mixes only with already heated fluid with strong buoyant forces and hence maintains it newly gained velocity. When the Re is large the resistance to the vertical fluid motion is reduced and hence the fluid vertical velocity at every point of inlet of the enclosure remains relatively high: Buoyant forces are much more dominant allowing fluid to rise more rapidly at the inlet. The vertical velocity therefore increases as the Reynolds number decreases. This is as a result of reduced vicious effects on the vertical motion of the fluid.

Table 3: Velocity for Varying Richardson Number at $Re = 50$

Height (h_i)	Ri=2	Ri=5	Ri=10
0	0.3660245	0.9150613	1.830123
1	0.4640981	1.160245	2.32049
2	0.4903678	1.225919	2.451839
3	0.497373	1.243433	2.486865
4	0.49911243	1.247811	2.495622
5	0.49911243	1.247811	2.495622
6	0.497373	1.243433	2.486865
7	0.497373	1.243433	2.486865
8	0.497373	1.243433	2.486865
9	0.497373	1.243433	2.486865

The results in the table 3 above is represented graphically as seen in figure 4 below

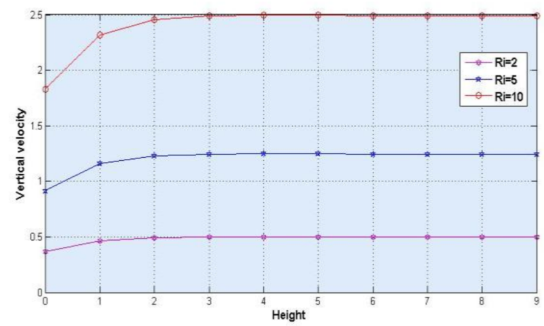

Figure 4: Vertical Velocity Graph against Enclosure Height when Varying the Ri Number

As the height of the inlet port is increased, velocity increases for a particular value of Ri as shown in Table 3 and Figure 4. This is because the buoyancy forces created by density differences are high near the heat source (heated wall).The velocity of the descending fluid is low near the cold walls. This is due to the mixing of warm and cold air resulting into low movement of fluid particles; hence low velocity. When varying Ri number, the velocity is seen to increase with increase in Ri number as evidenced in Table 3, Figure 4. This can be attributed to the fact that when the Ri number is large the buoyancy forces become comparatively large and the fluid velocity increases. At larger values of Ri and low Re the inertial force become negligible and the flow is governed by natural convection. As Ri increases across the table, velocity increases significantly due to large buoyancy forces forcing fluid particles to move at higher velocities.

4.3. Effects of Reynolds Number on Temperature Distribution

Table 4: Temperature for Varying Richardson number at $Pr = 0.71$

Height (h_i)	Re=50	Re=100	Re=200
0	34.0989	33.20438	32.60438
1	39.09341	37.83108	36.83108
2	40.8626	39.4579	38.5579
3	41.48927	40.24494	39.24494
4	41.71024	40.26791	39.26791
5	41.78289	40.27258	39.27258
6	41.77687	40.26388	39.27388
7	41.77546	40.27376	39.27376
8	41.77499	40.27236	39.27236
9	41.7733	40.27125	39.27125

The results in the table 4 above is represented graphically as seen in figure 5 below

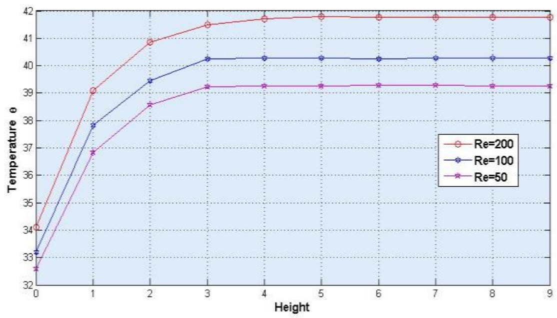

Figure 5: Temperature Graph against Enclosure Height when Varying the Re Number

As the height of the inlet port is increased, the temperature within the closure also increases for a particular value of *Re*. At the bottom of the enclosure the temperatures are comparatively lower due to increased distance from the vertical wall and due to the fact that much of air at the bottom is cold. As the height increases, there is a temperature increase after the fluid particles pass along the heated wall. The temperature of the enclosure remains almost constant as fluid is completely heated and the inlet positions varies from the bottom of the enclosure to the top. Rapid increase in temperature is only recorded at the base of the enclosure. As *Re* increases, temperature reduces as a result of reduced thermal boundary layer thickness near the heated wall.

V. Conclusions and Recommendations

5.1. Conclusion

A computational study is performed to investigate the mixed convection in a rectangular enclosure with the constant flux heated wall. Results are obtained for wide ranges of Reynolds number (*Re*) and Richardson number (*Ri*).

The following conclusions may be drawn from the present investigations:

- The forced convection parameter *Re* has a significant effect on the flow and temperature fields. Inertia force increases velocity profiles and thermal layer near the heated surface become thin and concentrated with increasing values of *Re*. The maximum temperature of the fluid is found to be optimum for the lowest *Re* as well as *Ri*.
- The cooling efficiency increases with increase in Reynolds and Richardson numbers.
- The best inlet location to obtain minimum temperature inside the enclosure is at the bottom left corner of the vertical wall.

5.2. Recommendations

Further work is recommended to improve on the results so far obtained for cooling effectiveness in an enclosure. This may be done by:

i. The fluid flow is considered to be non-laminar and steady, Re > 2300.
ii. The fluid inside the enclosure is assumed to be other fluids other than air.

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