Voltage Stability Assessment in DC Railways with Minimum Headway Consideration

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***Abstract*—**Voltage stability assessment is a crucial tool for headway planning in railway system operation. The use of appropriate minimum headway is an important factor that gives the metro system running fast and efficient while the system must have strong voltage stability. In view of the seriousness of this problem, the effective measure of voltage stability assessment should be taken for analyse the effect of train headway in DC railways. This paper describes the impact of minimum headway on voltage collapse of DC mass transit systems. This assessment is based on eigen-value sensitivity analysis of two voltage level systems. The smallest eigen-value is chosen to measure the stability margin of the DC railway mass transit system. As a result, the minimum headway of 3 minutes caused the voltage collapse of the 800 V test system while the 1.5 kV test system is still strong. The simulation result of all case shows that the 1500 V test system is stronger than 800 V.

***Keywords*—**DC Railway; Minimum Headway; Smallest Eigen-Value; Voltage Collapse; Voltage Stability.

I. Introduction

VOLTAGE instability is a phenomenon which often contributes to the development of power system disturbances. While increasing load power, node voltage decreases dramatically. As a result, voltage collapses with all consequences resulting from it. The problem of voltage stability concerns the whole power systems in both AC and DC sides, although it usually has a large involvement in one critical side of the power system. IEEE defines voltage collapse as the process by which voltage instability leads to loss of voltage in a significant part of the power system [IEEE/CIGRE Joint Task Force Report, 2004]. In DC power systems, analysis of DC voltage collapse differs from that of AC power systems in which reactive power plays the key role of impact to the AC voltage stability. In DC where the reactive power disappears there must be another approach to pin out the point of voltage collapse. Nowadays, in some countries especially in Asia there is a need for mass transit railways to mobilize their people within metropolitan area [Haque, 2003]. Due to sudden and frequent changes hungrily in power consumption of metro DC railway substation, the relation between train traffic operation and the voltage collapse must be carefully studied.

System analysis within the DC traction power system is vital to the design and operation of an electrified DC railway [Hill, 1994]. The DC traction-power simulators usually include modelling of the track geometry and traction-system characteristics and permit multi-train operations. By solving the DC power network equations, the simulators give details of electrical interactions among trains at specific time steps over a long span of time and under different traffic conditions. All moving loads in the DC railway power feeding system are assumed to be fixed in position at a specific time given. However, in the real world, the running trains change their position at every second. Hence, a full hour operation of train services is required to investigate the effect of minimum headway on the DC voltage stability.

This paper organizes a total of six sections. Next section, Section 2, illustrates DC railway power supply systems. Section 3 gives the brief of traction performance and train movement simulation. Section 4 presents the eigen-value sensitivity analysis to find the point of voltage collapses in DC railway power systems. Section 5 is the section describing simulation results and discussion. Conclusion remark is in the last section, Section six.

II. DC Railway Power Flow Calculation

A DC mass transit railway power system is a typical 3rd-rail conductor system. It is complicated as described in figure 1 for its conductor arrangement. To formulate power flow equations, transmission lines and other network components

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requires sufficiently accurate modeling. Analysis of DC railway power supply systems is simple and typically based on only nodal analysis [Goodman, 1997]. However, in more accurate model of the DC mass transit system, trains operating in powering mode draw a huge amount of power from the traction substation. It is reasonable to characterize the motoring train as a constant power model. Therefore, the DC railway power feeding systems are non-linear [Lee & Han, 2012; Saikong & Kulworawanichpong, 2014]. There are some simplifications of the DC power network as summarized in figure 2. This equivalent circuit is adequate to calculate voltages across trains and terminal voltage of the substations.

In this paper, only DC power flow methods based on the nodal analysis is determined. According to the Newton-Raphson method [Chong & Zak, 2001; Kulworawanichpong, 2010], the current mismatch equations of node m can be shown in (1). (2) describes the system matrix of the nodal analysis for the DC railway system [Hong, 2007]. It notes that the generator current mentioned above is the currents supplied by the supply sources at the substations which are constant, J1S and J2S. The load current is the current drawn by the powering train. It can be expressed mathematically as summarized in (3).

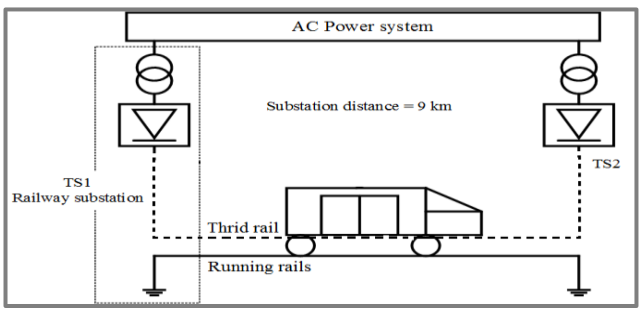

Figure 1: Simplified Railway System Interface

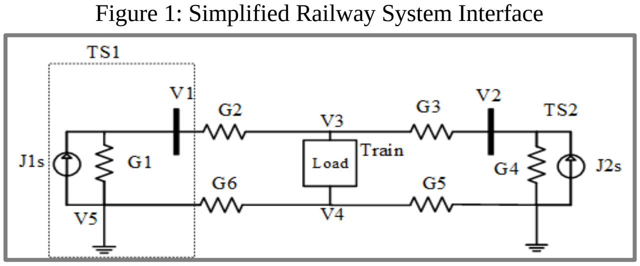

Figure 2: Norton Equivalent Circuit

$$I_{Gi} - I_{Di} = \sum_{j=1}^{n} G_{ij}\left(V_i - V_j\right) \tag{1}$$

Where

I_{Gi} is the generator current of node i

I_{Di} is the load current of node i

G_{ij} is the conductance between node i and node j

$$\begin{bmatrix} G1+G2 & 0 & -G2 & 0 \\ 0 & G3+G4 & -G3 & 0 \\ -G2 & -G3 & G2+G3 & 0 \\ 0 & 0 & 0 & G5+G6 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} = \begin{bmatrix} J1s \\ J2s \\ -I_{Di} \\ I_{Di} \end{bmatrix} \tag{2}$$

$$I_{Di} = \frac{P_{train}}{V_3 - V_4} \tag{3}$$

Where P_{train} is the power consumed by the powering train.

III. Traction Performance Calculation

The powering train modelling used the train power consumption from tractive effort force-speed characteristic for propulsion shown in figure.3. The power train consumption by tractive force multiplying with velocity is described in (4).

$$P_{train} = F_T v \tag{4}$$

Where

P_{train} is train's power consumption

v is train's velocity

F_T is train's tractive effort

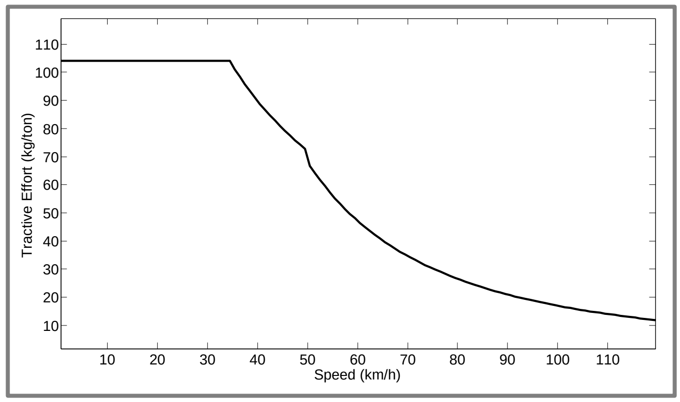

Figure 3: Tractive Effort Characteristic for Propulsion

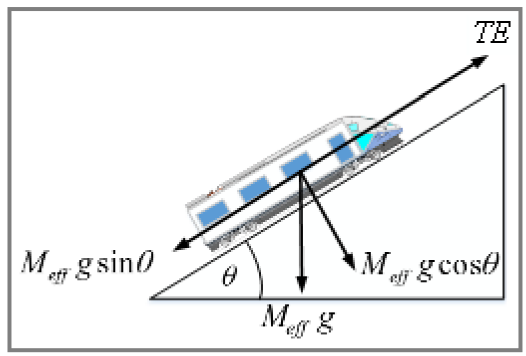

Figure 4: Free-Body Diagram of a Train on a Gradient

Suppose that a train of mass M_{eff} is on a slope making an angle θ to the horizontal as shown in figure 4, where M_{eff} is the effective mass. The train motion is opposed by various forces, e.g. aerodynamic drag, track gradient force, etc. By applying Newton's second law, the train movement equation is expressed in (5), where α is the train acceleration [The Electric Railway Handbook Editorial Committee, 2007].

$$TE - F_{grad} - F_{drag} = M_{eff}\alpha \tag{5}$$

The aerodynamic drag is difficult to predict from calculations, however based on measurements from run-down tests where the natural deceleration of a train on straight, level track on a windless day is measured the drag force can be characterized. Different operators have their own favourite equation to fit the train resistance. The Davis in (6) as is commonly used.

$$F_{drag} = a + bv + cv^2 \tag{6}$$

Where a, b and c are drag-force coefficients and v is the train speed. It is common to use different values for open or tunnel situations [Goodman, 1997; Profillidis, 2006]. To push a heavy train up slopes requires substantial force. The gradient force can be approximated by using (7).

$$F_{grad} = M_{eff}\, g \sin\theta \tag{7}$$

The friction force is calculated using train resistance [Kulworawanichpong, 2010], show in (8).

$$R_T = R_R + (R_G + R_C + R_{Tu}) \tag{8}$$

Where

R_T is Total of Train resistance (N)
R_R is Rail Resistance (N)
R_G is Gradient Resistance
R_C is Curve Resistance
R_{Tu} is Tunnel Resistance

The rail train resistance is calculated in (9).

$$R_R = \left(f_0 + f_1 V\right)W + f_3 V^2 \tag{9}$$

Where

V is train's speed (km/h)
W is Train mass (ton)
f_0, f_1 are rolling resistance coefficients
f_3 is an aerodynamic drag coefficient

IV. Eigen-Value Sensitivity Analysis

The eigen analysis is an approach to measure proximity of Jacobian singularity at loadability limits [Kundur, 1994; Cutsem & Vournas, 1998]. Eigen-value decomposition can be expressed in terms of eigen-values and eigenvectors as

$$A = W_\sigma \Sigma V_\sigma^T \tag{10}$$

Where V_σ and W_σ are matrices whose columns are the right and left eigenvector respectively, while Σ is a diagonal matrix whose entries are the eigen-values σ_i $(i = 1,\ldots n)$ of A. The eigenvectors are such that

$$Av_{\sigma i} = \sigma_i w_{\sigma i} \tag{11}$$

$$w_{\sigma i}^T A = \sigma_i v_{\sigma i}^T \tag{12}$$

For a singular Jacobian matrix, the smallest eigen-value is zero as follows.

$$Av_{\sigma i} = 0 \tag{13}$$

$$w_{\sigma i}^T A = 0 \tag{14}$$

In this paper the smallest eigen-values analysis is provided for study of voltage stability. The network constraints represented by equation (1) may be expressed in following form

$$[\Delta I] = [J_{PV}][\Delta V] \tag{15}$$

Where *is* ΔI incremental change in node current, ΔV is incremental change in node voltage magnitude and $[J_{PV}]$ is the Jacobian matrix. The power flow equations are solved using the Newton-Raphson technique [Pai, 2006; Ajjarapu, 2006]. The magnitude of the eigen-values of the Jacobian matrix in equation (15) can provide a relative measure of proximity to instability.

V. Simulation Results

This section aims to present the simulation results of different headways and different voltage level to exhibit voltage stability of DC metro railway systems. The test is a DC railway system of 9-km line route having three passenger stops with equal 3-km passenger station spacing. Figure 5 shows the train speed trajectory. Figure 6, 7 and 8 show the sum of train power consumption for this test.

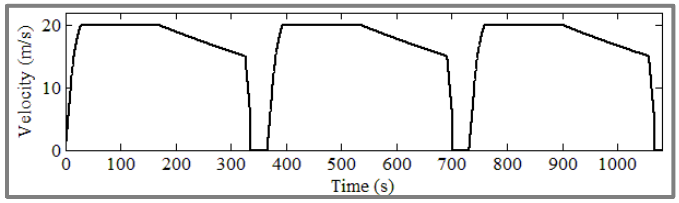

Figure 5: Train Speed Trajectory of the Test

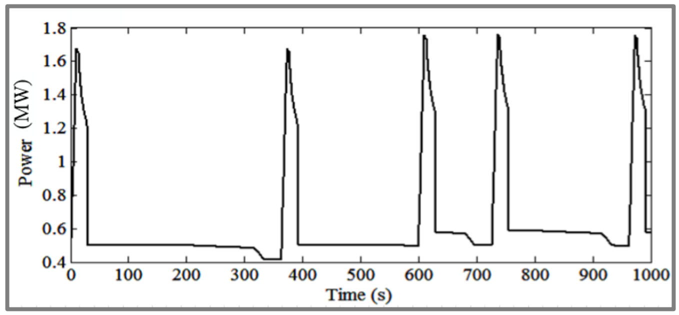

Figure 6: The Sum of Train Power Consumption for 10-Minute Headway

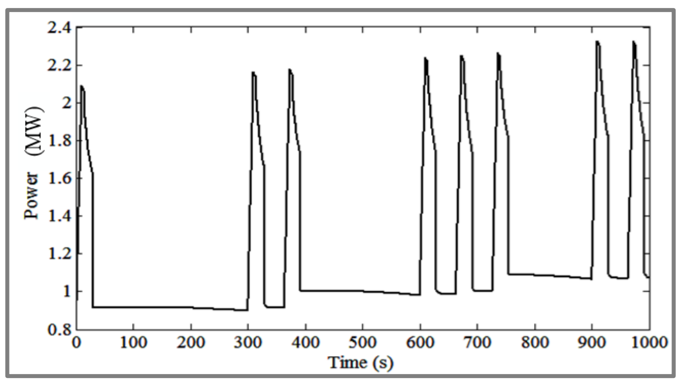

Figure 7: The Sum of Train Power Consumption for 5-Minute Headway

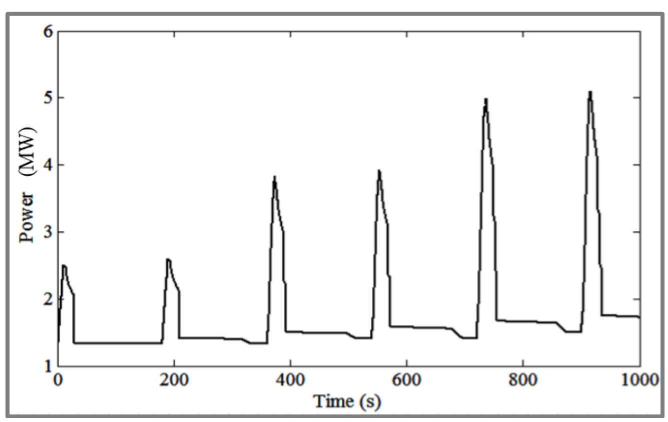

Figure 8: The Sum of Train Power Consumption for 3-Minute Headway

The test is divided into three test case scenarios according to three different headways and two different voltage level test systems. The first test case scenario is the 10-minute headway. The train travel graph generated according to the 10-minute headway is shown in figure 9. To measure the voltage stability of this test case scenario, the smallest eigen-values are computed and collected with respect to time. Figure 10 and 11 show the smallest eigen-value at each time step during the train service of 800 V and 1.5 kV test systems. The minimum smallest eigen-value of this test is 310.43 for 800 V test system and 158.40 for 1.5 kV test systems respectively. This exhibits that the first test case scenario is in the safety zone and the voltage collapse does not occur. The voltage magnitude of the substation terminal 1 of both voltage level test systems are shown in figure 12 and 13 respectively.

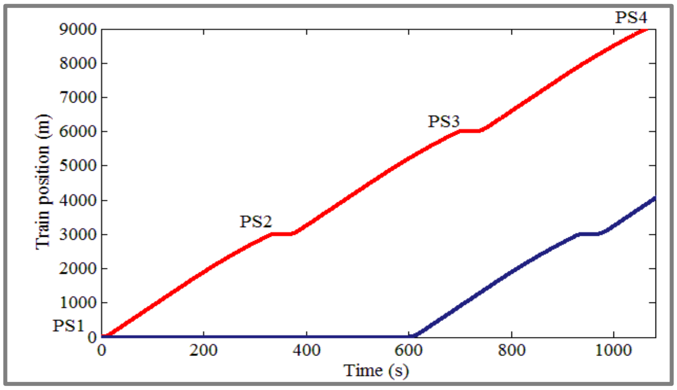

Figure 9: Train Travel Graph for 10-Minute Headway

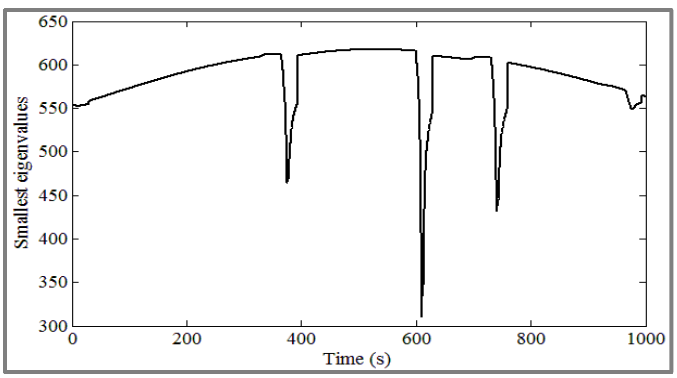

Figure 10: Smallest Eigen-Value of 800 V System for 10-Minute Headway

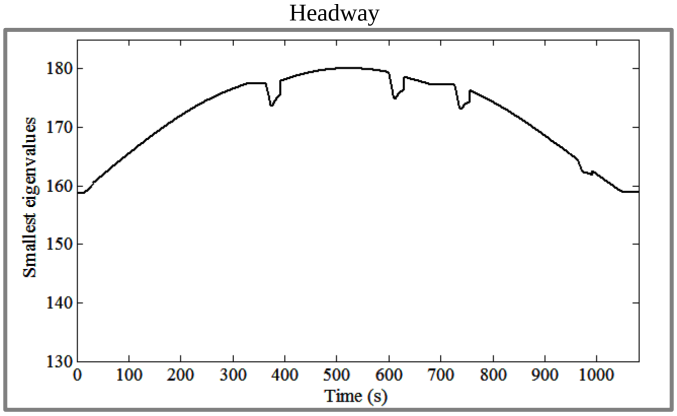

Figure 11: Smallest Eigen-Value of 1.5 kV System for 10-Minute Headway

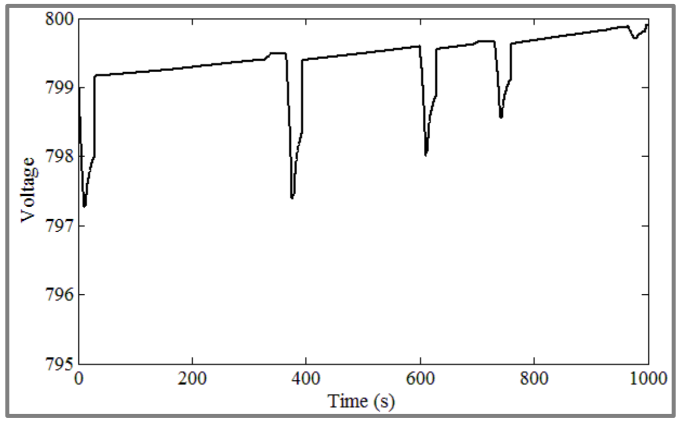

Figure 12: Voltage of 800 V Test System at the Substation Terminal 1 for 10-Minute Headway

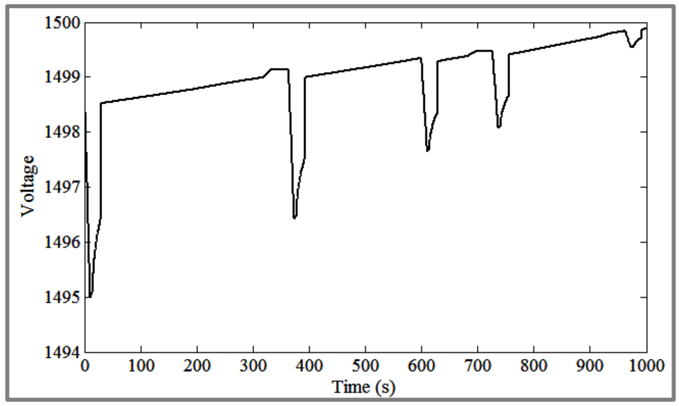

Figure 13: Voltage of 1.5 kV Test System at the Substation Terminal 1 for 10-Minute Headway

The second test case scenario is the 5-minute headway. The train travel graph generated according to the 5-minute headway is shown in figure 14. To measure the voltage stability of this test case scenario, the smallest eigen-values are computed and collected with respect to time. Figure 15 and 16 show the smallest eigen-value at each time step during the train service. The minimum smallest eigen-value of this test is 8.71 for 800 V test system and 158.80 for 1.5 kV test systems respectively. This exhibits that the second test case scenario is still in the safety zone and the voltage collapse does not occur. The voltage magnitude of the substation terminal 1 for this test case scenario is shown in figure 17 and 18.

The third test case scenario (the last case) is the 3-minute headway. The train travel graph generated according to the 3-minute headway is shown in figure 19. To measure the voltage stability of this test case scenario, the smallest eigen-values are computed and collected with respect to time. Figure 20 and 21 show the smallest eigen-value at each time step during the train service. The minimum smallest eigen-value of 800 V test system is 0. This indicates that the last test case scenario is unstable and the voltage collapse does occur, while the minimum smallest eigen-value of 1.5 kV test system is 148.85 that mean the case scenario of 1.5 kV test system is still in the safety zone and the voltage collapse does not occur. The voltage magnitude of the substation terminal 1 for this test case scenario is shown in figure 22 and 23.

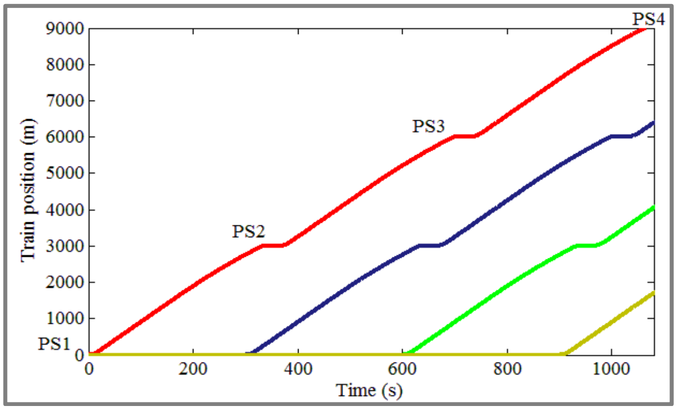

Figure 14: Train Travel Graph for 5-Minute Headway

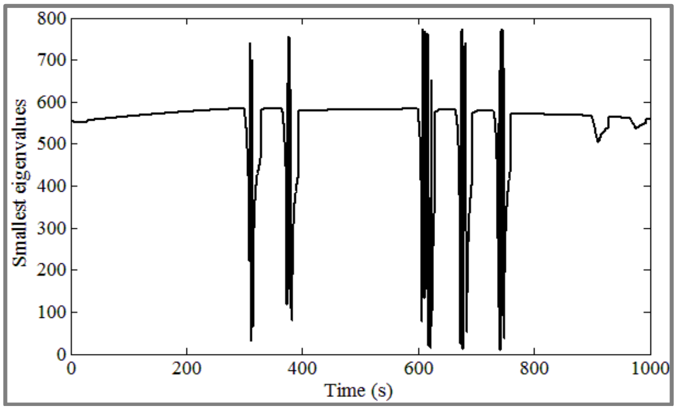

Figure 15: Smallest Eigen-Value of 800 V System for 5-Minute Headway

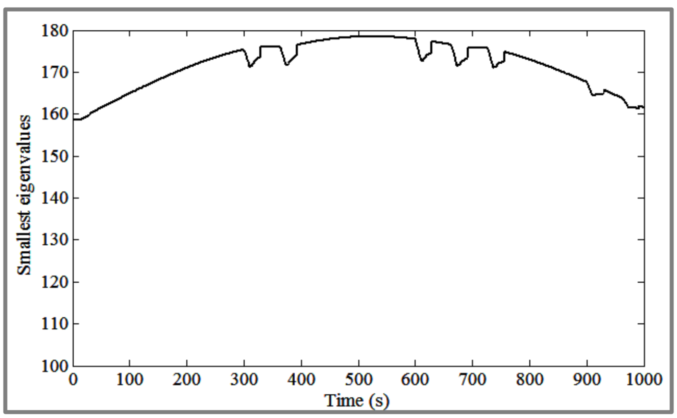

Figure 16: Smallest Eigen-Value of 1.5 kV System for 5-Minute Headway

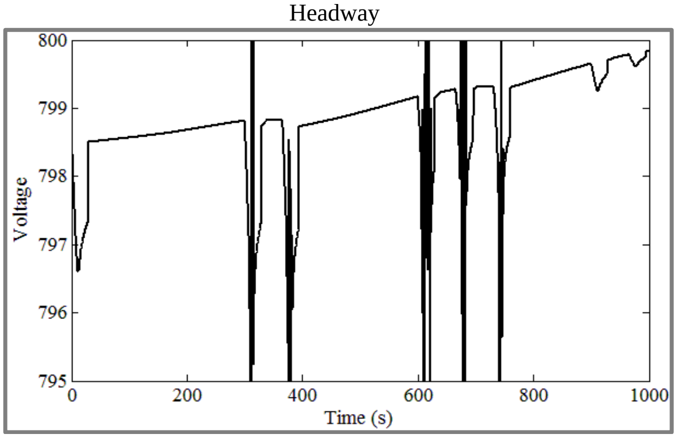

Figure 17: Voltage of 800 V Test System at the Substation Terminal 1 for 5-Minute Headway

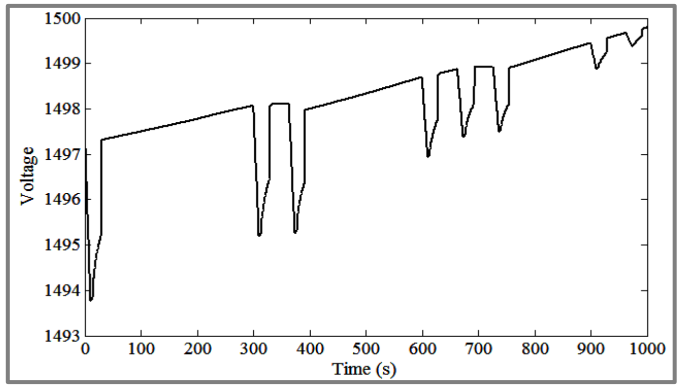

Figure 18: Voltage of 1.5 kV Test System at the Substation Terminal 1 for 5-Minute Headway

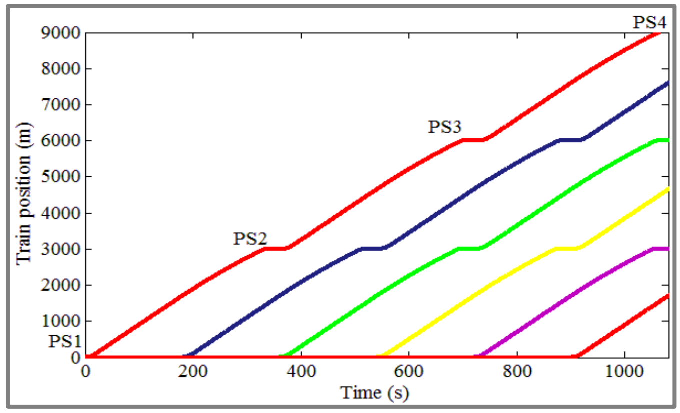

Figure 19: Train Travel Graph for 3-Minute Headway

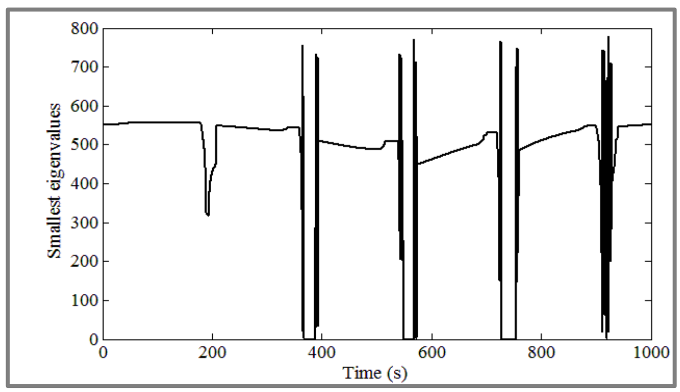

Figure 20: Smallest Eigen-Value of 800 V System for 3-Minute Headway

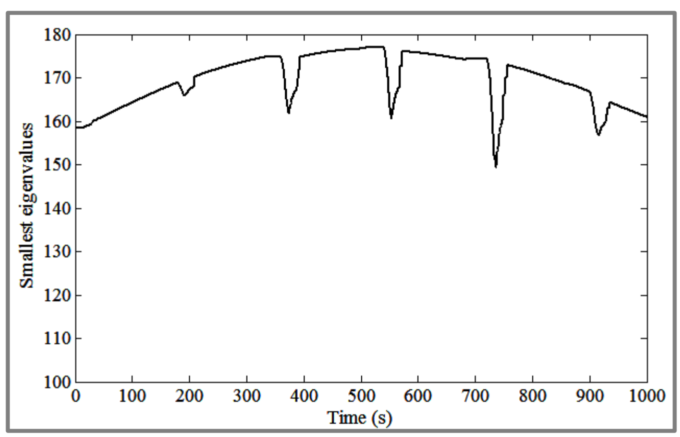

Figure 21: Smallest Eigen-Value of 1.5 kV System for 3-Minute Headway

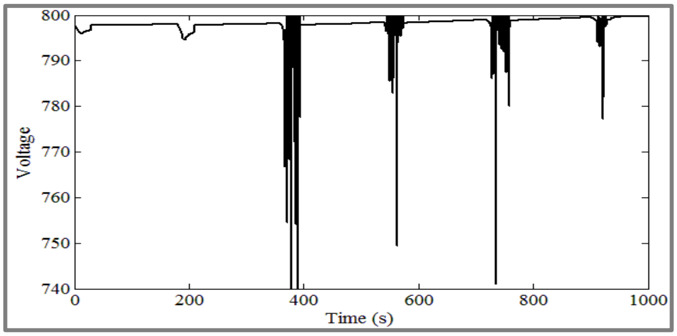

Figure 22: Voltage of 800 V Test System at the Substation Terminal 1 for 3-Minute Headway

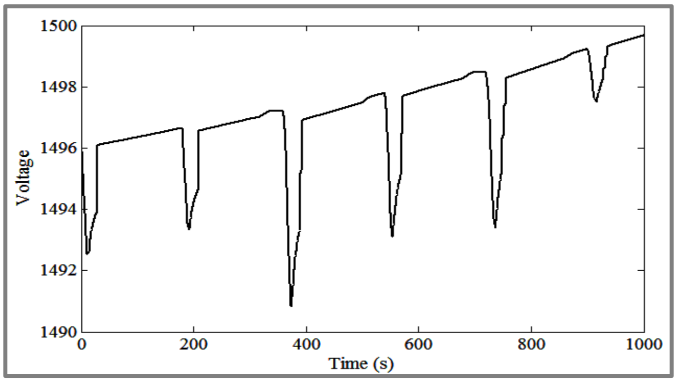

Figure 23: Voltage of 1.5 kV Test System at the Substation Terminal 1 for 3-Minute Headway

VI. Conclusion

This paper describes the method of smallest eigen-value to measure voltage stability of DC metro railway power systems. The eigen-value calculation is based on the Newton-Raphson iterative method to formulate Jacobian matrices. The smallest eigen-value as mentioned is the eigen-value of these Jacobian matrices. A DC metro railway system of 9-km line route having three passenger stops with equal 3-km passenger station spacing is employed for test. According to the simulation results, as smallest eigen-values, voltage stability is reduced when the headway decline. As a result of all test case scenarios, the trends in all cases are similar but the 1500 V test system is stronger than 800 V and the minimum headway of 800 V test system for 3 minutes is the case that causes the system voltage collapse. Because of the power loss in high voltage system is less than the low voltage when used same load.

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