

Synchronization of Parallel Dual Inverted Pendulums using Optimal Control Theory

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Abstract—The inverted pendulum which is used as a benchmark for implementing the control methods, is a highly nonlinear unstable system. In this paper the modeling and control design of nonlinear inverted pendulum-cart dynamic system is presented. A partially linearized model for the under actuated Euler-Lagrangian system composed of two single linear inverted pendulums is obtained based on the feedback linearization method. Then, the small deviation linearization technique is applied to the partially linearized model to obtain the linear model. For the linear model which is completely controllable, we propose a method to construct a synchronization error signal between the two inverted pendulum systems to guarantee that an augmented system, which contains the original state variables of the two subsystems and the synchronization error, is still completely controllable. For the augmented system an optimal synchronization controller is designed. Experimental results show that the optimal synchronization control system has been successful in commanding the pendulums to move in synchronized fashion.

Keywords—Euler-lagrangian; Feedback Linearization; Inverted Pendulum; Lyapunov Theory; Optimal Control; Riccati Equation; Synchronization Control.

Abbreviations—Control Moment Gyroscope (CMG); Inverted Pendulum (IP); Linear Quadratic Regulator (LQR); Pendulum On A Cart (POAC); Unmanned Aerial Vehicles (UAV).

I. INTRODUCTION

IN motion control systems a basic topic is to keep several moving objects in synchronization. There extensively exists such a topic in mechanical systems, electro-mechanical systems, robots, spacecraft and the chaotic systems [Tomizuka et al., 1992; Sun, 2003; Sun & Mills, 2002; Liu & Shan, 2005; Yau et al., 2005]. According to the basic structure of synchronization control systems, the master-slave and cross-coupling approaches are usually employed to realize synchronization. However, there inevitably exists a lag while the slaver is tracking the master by using the master-slave approach. If the dynamics of objects are similar, the master-slave control approach cannot achieve an ideal dynamical synchronization. In this case the cross-coupling control scheme should perform better, because a second lag is avoided. In the cross-coupling approach some synchronization errors are introduced to form some cross-coupling items between two or more unrelated motions.

About the cross-coupling synchronization control, Tomizuka et al., (1992), proposed an adaptive feed forward synchronization control strategy for the speed synchronization of two DC motors. In this article to obtain a synchronous motion of two DC motors whose dynamics are described as the lag terms, an adaptive coupling controller with synchronization error feedback was proposed. If one DC motor suffered a disturbance, the motion errors could be used to adjust the motion of the other one, and that improved the synchronization performance. Based on the Lyapunov design method, Sun (2002; 2003; 2003A) proposed an adaptive cross-coupling synchronization controller for Lagrangian nonlinear systems, which could be applied to the synchronization control of two or more objects. The proposed algorithm guaranteed the asymptotic convergence of the tracking error and the synchronization error. Liu & Sun (2005) presented a uniform motion synchronization strategy which achieved the asymptotic convergence of tracking error and synchronization error and improved the transient motion

performance. Furthermore, a general definition of synchronization error was given by them. Kamano et al., (1993) applied the adaptive feed forward synchronization control strategy to the position synchronization of two objects. Liu & Shan (2005) proposed an adaptive synchronization strategy and applied it to the attitude angular velocity tracking control of multiple Unmanned Aerial Vehicles (UAVs). It achieved a global asymptotic convergence for the attitude angular velocity tracking and the angular velocity synchronization, even in the presence of system parameter uncertainties. Tan et al., (1996; 1997) proposed an optimal synchronization position control scheme for a kind of discrete linear systems. The scheme was obtained by minimizing the linear quadratic performance index which involves both position and synchronization errors. Xiao et al., (2005) and Zhou et al., (2003) designed optimal synchronization controller for linear systems by augmenting the original systems with introducing a synchronization error. All above results are achieved for full actuated systems. Consequently, systematic synchronization control schemes have been established for full actuated Lagrangian systems. However, for the synchronization control of underactuated Lagrangian systems few have been studied. In practical applications, many control systems are underactuated and their synchronization control is important.

For the synchronization control of underactuated Lagrangian systems only some specific instances have been researched. Zhou et al., (2006) proposed a synchronization control law for twin-gyro CMGs which steer an open-loop slewing of a truss arm. Tsai & Shen (2007) presented a control approach to the synchronization control of a parallel dual inverted pendulums system (IPs). For each IP system an inner loop pendulum robust balance controller and an outer loop car position controller are designed based on the small deviation linearization models. Lal Bahadur Prasad et al., (2011) has designed an optimal controller to control the Pendulum On A Cart system (POAC) using LQR and PID controller. Dongfang Zhu & Di Zhou (2008) states that the designing approach is complicated and the synchronization accuracy is not very high. In that paper, research the synchronization control scheme for the under actuated Lagrangian system composed of parallel dual inverted pendulums.

This paper proposes a novel strategy to control and stabilize the inverted pendulum which is the base of (benchmark problem) of many fields, for e.g. ship stabilization, missile stabilization, etc. The aim of the paper is to synchronize the motion of two individual pendulums on their respective carts, to reduce the synchronization error, to have better dynamics than Dongfang Zhu & Di Zhou (2008). The second section of the paper linearizes the given nonlinear model of a single pendulum. The third section of the paper augments the two inverted pendulums into a single system; we also design the controller for the above system. The fourth section discusses the simulation results of the augmented system and shows the synchronization error of the two pendulums is very realistic and minimum.

II. MODELLING OF SINGLE INVERTED PENDULUM

Ignoring the air friction, we may think a single inverted pendulum as a device composed of a car and a rigid pendulum. On the basis of Lagrange formulation, the single inverted pendulum model can be described by the following nonlinear equations [Eltohamyk & Kuo, 1999]:

$$\begin{bmatrix} M+m & ml \cos \phi \\ -ml \cos \phi & 4ml^2/3 \end{bmatrix} \begin{bmatrix} \ddot{\xi} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ -mgl \sin \phi \end{bmatrix} + \begin{bmatrix} b & ml \dot{\phi} \sin \phi \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\xi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (1)$$

Where M is the mass of the car, m is the mass of the pendulum, l is the length between the rotation point and the centroid, b is the coefficient of friction between the car and its orbit, g is the acceleration of gravity, ξ is the position of the car, ϕ represents the rotation angle of the pendulum, and τ denotes the control force.

Investigating (1) and choosing a state vector as

$$q(t) = [q_1(t) \ q_2(t) \ q_3(t) \ q_4(t)]^T = [\xi(t) \ \phi(t) \ \dot{\xi}(t) \ \dot{\phi}(t)]^T$$

We have,

$$\begin{aligned} \ddot{\xi} &= f_1(q_2, q_3, q_4) + \frac{4g_1(q_2)}{3} \tau \\ \ddot{\phi} &= f_2(q_2, q_3, q_4) + g_2(q_2) \tau \end{aligned} \quad (2)$$

Where,

$$\begin{aligned} g_1(q_2) &= \frac{3}{4M+m+3m \sin^2 \phi} \\ g_2(q_2) &= \frac{\cos \phi}{l} g_1(q_2) \\ f_1(q_2, q_3, q_4) &= \left(\frac{mg \sin \phi \cos \phi - 4(ml \sin \phi \dot{\phi}^2 + b \dot{\xi})}{3} \right) g_1(q_2) \\ f_2(q_2, q_3, q_4) &= \frac{3 \cos \phi}{4l} f_1(q_2, q_3, q_4) + \frac{3g \sin \phi}{4l} \end{aligned}$$

And its form in state-space:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} q_3 \\ q_4 \\ f_1(q_2, q_3, q_4) \\ f_2(q_2, q_3, q_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4g_1(q_2)/3 \\ g_2(q_2) \end{bmatrix} \tau \quad (3)$$

For system (3) let the output $y = h(q) = q_1$. Assume that q_0 is an equilibrium point of system (1). In a neighborhood of q_1 , there exist $L_g h(q) = 0$, $L_f^2 h(q) = f_1(q_2, q_3, q_4) \neq 0$, $L_g L_f h(q) = 4g_1(q_2)/3 \neq 0$. According to the nonlinear system feedback linearization theory system [Isidori, 1995] has a relative Degree 2 at q_0 . Thus system (3) can just be partially feedback linearized. Let the state feedback control law be

$$\begin{aligned} \tau &= (L_g L_f h(q))^{-1} (-L_f^2 h(q) + v) \\ \tau &= -\frac{3}{4} mg \sin \phi \cos \phi + ml \sin \phi \dot{\phi}^2 + b \dot{\xi} + (4M+m+3m \sin^2 \phi) v / 4 \end{aligned} \quad (4)$$

Substituting control law (4) into system (3) yields,

$$\begin{bmatrix} \dot{\xi} \\ \dot{\phi} \\ \ddot{\xi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{\xi} \\ \dot{\phi} \\ 0 \\ \frac{3g \sin \phi}{4l} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{3 \cos \phi}{4l} \end{bmatrix} v \quad (5)$$

When the pendulum is stabilized at the inverted point, $\phi \leq 5^\circ$. Hence with the small deviation linearization technique we have $\sin \phi = \phi$ and $\cos \phi = 1$. Finally, the linearized model of a single inverted pendulum is given by,

$$\begin{bmatrix} \dot{\xi} \\ \dot{\phi} \\ \ddot{\xi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{3g}{4l} & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \phi \\ \dot{\xi} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3} \\ \frac{3}{4l} \end{bmatrix} v \quad (6)$$

III. OPTIMAL FEEDBACK SYNCHRONIZATION CONTROLLER

Integrating state equations of two single inverted pendulums together gives,

$$\begin{aligned} \ddot{\phi}_1 &= \frac{3g\phi_1}{4l_1} + \frac{3}{4l_1} v_1 \\ \ddot{\phi}_2 &= \frac{3g\phi_2}{4l_2} + \frac{3}{4l_2} v_2 \\ \ddot{\xi}_1 &= v_1 \\ \ddot{\xi}_2 &= v_2 \end{aligned} \quad (7)$$

In the following text, we consider the car's motion of inverted pendulum device 1 as Axis 1 motion, denoted by ξ_1 , and the car's motion of device 2 as Axis 2 motion, denoted by ξ_2 . When Axis 1 and Axis 2 track the same command ξ_d , the position tracking errors of both the axes are defined as $e_1 = \xi_1 - \xi_d$, and $e_2 = \xi_2 - \xi_d$, respectively. With (7) the tracking error motion equations of the under actuated system are written as,

$$\begin{aligned} \ddot{\phi}_1 &= \frac{3g\phi_1}{4l_1} + \frac{3}{4l_1} v_1 \\ \ddot{\phi}_2 &= \frac{3g\phi_2}{4l_2} + \frac{3}{4l_2} v_2 \\ \ddot{e}_1 &= v_1 - \ddot{\xi}_d \\ \ddot{e}_2 &= v_2 - \ddot{\xi}_d \end{aligned} \quad (8)$$

Define the position synchronization error between Axis 1 and Axis 2 as $\varepsilon = e_1 - e_2$, and choose a synchronization state variable, $x_1 = \int_0^t \varepsilon dt = \int_0^t (e_1 - e_2) dt$, augmenting system (8) with x_1 results in,

$$\dot{X} = AX + B[v_1 \quad v_2]^T$$

Where,

$$X = [x_1 \quad \phi_1 \quad \phi_2 \quad e_1 \quad e_2 \quad \dot{\phi}_1 \quad \dot{\phi}_2 \quad \dot{e}_1 \quad \dot{e}_2]^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{3g}{4l_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3g}{4l_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{3}{4l_1} & 0 \\ 0 & \frac{3}{4l_2} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (9)$$

For system (9), choose the following integral quadratic performance index:

$$J = \int_0^\infty (X^T Q X + u^T R u) dt \quad (10)$$

Where Q and R are respectively symmetric semi-positive definite and positive definite matrices.

Let the feedback control vector be,

$$u = -KX \quad (11)$$

Where K is the feedback gain matrix. As the pair (A,B) is completely controllable, we can obtain an optimal feedback matrix K.

$$K = R^{-1} B^T P \quad (12)$$

Where K is the positive definite solution of the following algebraic matrix riccati equation:

$$A^T P + PA - PBR^{-1}B^T P = -Q \quad (13)$$

Construct a Lyapunov function for system (9):

$$V = \frac{1}{2} X^T P X \quad (14)$$

Differentiating V gives,

$$\begin{aligned} \dot{V} &= \frac{1}{2} X^T (A^T P + PA - 2PBR^{-1}B^T P) X \\ &= -\frac{1}{2} X^T Q X \leq 0 \end{aligned} \quad (15)$$

Then, $X \rightarrow 0$, $x_1 = \int_0^t \varepsilon dt \rightarrow 0$, as $t \rightarrow \infty$. According to Barbalat lemma, there must be $\varepsilon \rightarrow 0$ i.e. the synchronization errors tend to zero. Since $e_1 \rightarrow 0$, $e_2 \rightarrow 0$ as $t \rightarrow \infty$ there exists $\xi_1 \rightarrow \xi_d$, $\xi_2 \rightarrow \xi_d$.

In the above design both the tracking errors of each control axis and the integral of synchronization error are evaluated that guarantees the transient performance of each control axis and the synchronization performance simultaneously.

Substituting the state feedback control law (11) into feedback linearization control law (4) gives the synchronization control law of the system.

IV. EXPERIMENTAL RESULTS

The dual inverted pendulums used for synchronization experiment are shown in fig (1). The two devices are of the identical structure and they can be modeled with the same parameters: $M=1.096\text{kg}$, $m=0.109\text{kg}$, $l=0.25\text{m}$, $b=0.1\text{N/ms}^{-1}$, $g=9.82\text{ m/s}^2$.



Figure 1: Dual Inverted Pendulums used for Synchronization Experiment [Dongfang Zhu & Di Zhou, 2008]

The aim of the experiment is that the pendulums should follow the given command (reference) signal. The two pendulums should track the given reference signal in a synchronized fashion. The reference is a square signal with 0.1m gain and 10s periods.

To apply the optimal synchronization control to system, the weighting matrixes are chosen as,

$$Q = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.001 \end{bmatrix}$$

$$R = I_2$$

By solving the equation (13) the feedback gain matrix is obtained:

$$k = \begin{bmatrix} -1.2628 & 25.6434 & -4.4783 & -2.2504 & 2.1473 & 4.1038 & -0.7145 & -2.2373 & 1.7407 \\ 1.2327 & -4.5064 & 25.6451 & -2.1713 & -2.2731 & -0.7169 & 4.092 & 1.7502 & -2.2475 \end{bmatrix}$$

Substituting the linear feedback control law into (4) yields a stable balance control law. Inverted pendulum device 1 and 2 use the same control law.

Fig (2) shows the response of the two pendulums tracking the command signal. Both carts are oscillating around the origin with a radius of 0.1m. The actual position shows some error when the reference signal is changing directions. The tracking response and the synchronization error are shown in Figs. (2), (3) and (4). We see that the pendulums track the reference signal effectively and the lag between the pendulums is minimum.

It is observed from Fig (3) that pendulums 1 and 2 have reached synchronization with the maximal synchronization error are 0.00008m. Fig (4) shows the vertical angle of the two pendulums. We see that the angle is in milli range (nearly zero), which means that the pendulums are in upright position. The experimental results exhibits that the performance of the optimal synchronization control has been better than that of the synchronization scheme in Tsai & Shen (2007) in which the maximal synchronization error was 0.025m. In Tsai & Shen (2007), a disturbance was injected into the system when the cars' positions did not track a command. The optimal synchronization control system exhibits a mutual coordination and a mutual constraint between the controlled axes.

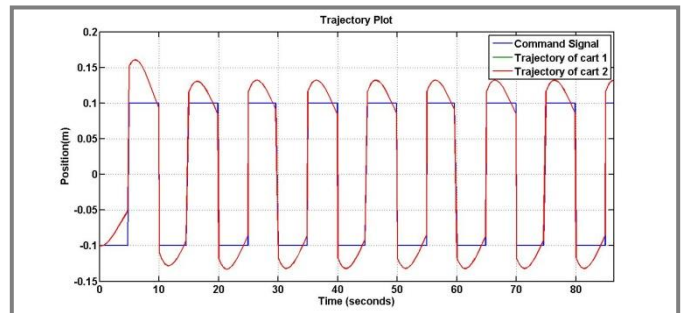


Figure 2: Trajectory of the Pendulums

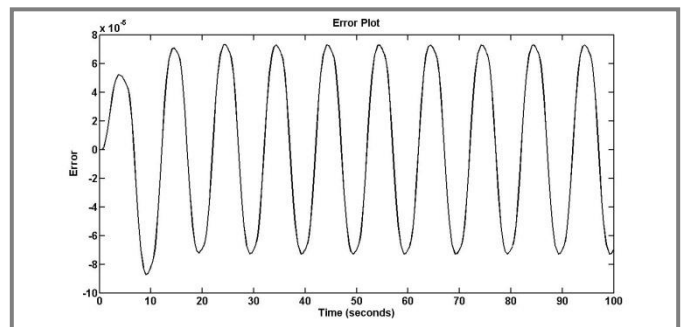


Figure 3: Synchronization Error Plot

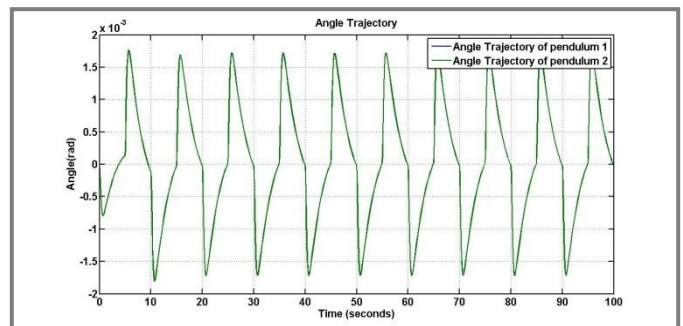


Figure 4: Angle Trajectory of the Pendulum

V. CONCLUSION

This paper investigates the synchronization control of dual inverted pendulum control system. The motion of a single inverted pendulum has been described as a linear model via using the partially feedback linearization and small deviation linearization. The integral of synchronization error of the controlled axes has been introduced to make an augmented

system which has involved the original state variables of the dual inverted pendulum system and the synchronization error. The augmented system was completely controllable, and so the optimal synchronization controller has been designed. It has satisfied the command tracking requirement and the synchronization accuracy. The pendulums have been successfully stabilized on their respective carts, while the carts are commanded to follow the given reference signal. The synchronization in the movement of the pendulums and their carts has been fairly achieved. Although there are many applications, one of the main uses of this research is to synchronize the robotic limbs using this theory. Also disturbances have not been considered and more emphasis has been given on the optimization of error, which can be the future scope of this paper.

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