

Robust Tracking of Linear Systems for Matched Uncertainty using LQR Approach

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Abstract—Robust control is derived for general linear uncertain systems with scalar control. In this paper we have shown way to solve a robust control problem of a uncertain system (matched uncertainty) by transforming it into an optimal control problem which becomes Linear Quadratic Regulator problem (LQR) for linear systems and can be solved by using algebraic Riccati equation. We assume that the control objective is not only to drive the states to zero but also to track a non-zero reference signal which is to be assumed as polynomial function of time. We also emphasized on finding the largest value of uncertain parameter for which given system is on the verge of instability, which has not been mentioned in the previous work. The control is derived by using LQR method and has state negative feedback form. The performance of proposed controller is examined by simulating linear system model in MATLAB environment.

Keywords—HJB Equation; LQR Problem; Lyapunov Function; Matched Uncertainty; Optimal Control; Riccati Equation; Robust Control; Tracking Problem.

Abbreviations—Linear Quadratic Regulator (LQR); Integral Linear Constraints (ILC's); Hamilton-Jacobi-Bellman (HJB); Linear Time Variant (LTV); Multi Input Multi Output (MIMO).

I. INTRODUCTION

THE early methods of Bode and others in control theory were fairly robust. The state space models invented in 1960s and 1920s were sometimes found to lack of robustness, prompting to improve them which were the start of the robust theory. Robust control is a limb of control hypothesis that expressly manages uncertainty in its approach to controller design. Robust methods aim to achieve robust performance or stability in the presence of bounded modelling errors. In real life, the systems to be controlled have many uncertainties that can deviate the system performance from the desired. So designing effective robust controllers for such systems becomes significant. In the past, there have been many works on robust control design of uncertain systems, but the work focused on the robust regulator problem.

In Barmish (1985), necessary and sufficient conditions for quadratic stability of unforced system and then controlled system are determined for the state and the output feedback case. In that linear time varying systems subjected to norm bounded, structured uncertainties considered. Barmish et al., (1983) emphasized on the problem of designing a stabilizing controller for a class of uncertain dynamical systems. The

stability and control of uncertain systems using co-positive linear Lyapunov function along with dissipativity theory is done by Corentin Briat. The Integral Linear Constraints (ILC's) are used for the robust analysis. For the class of robust stabilization problems the construction of universal controller is discussed by Battilotti (1997). General theorems on the construction of the universal controllers were discussed, which requires that a certain non-linear inequality is solvable or equivalently that a robust control Lyapunov function does exist. A type of observer called the Proportional-Integral (PI) observer proposed for the design of the robust controller by Beale & Shafani (1989). In Amato et al., (1996), linear time-varying systems subject to norm bounded, one block, structured uncertainties presented. This observer contrasts from the ordinary one by an integration path which gives extra degree of freedom [Beale & Shafani, 1989]. Whereas augmented systems approach is used for the measurement based robust tracking problem in Benson & Schmitendorf (1997). The robust optimal control of linear uncertain system with L_2 -bounded uncertainties is discussed in Chen (1993). The robust tracking control problem for a class of partial information MIMO LTV systems using multi-linear model approximation approach is employed because only a finite number of discrete characteristic time points

during the whole operating region of the system could be measured is presented in Lei Song et al., (2010).

For decentralized robust control of a class of large- scale uncertain dynamical system using deterministic design approach proposed by Chen (1988). The two types of robust control algorithm are proposed, namely local control and the global control. The local control utilizes the local state of each subsystem as feedback while global control uses local state as well as states of neighboring subsystems as the feedback. In Dawson et al., (1990), the stability of a PD controller is examined for the trajectory tracking problem of robot manipulator. A Lyapunov’s second method is used to derive a uniform boundness for the PD controller. The likelihood of deciding a linear control law when the full state can't be measured and observers are designed to estimate the state is recognized in Faryar & Schmitendorf (1991). The proposed control law and the observer are designed using two Riccati equations. Focus was on systems which satisfied matched uncertainty condition. A class of non-linear uncertain systems is considered for the stabilization using polynomial robust control in which the bound of uncertainty does not need to be known [Han & Chen, 1992]. Leitmann (1979) discusses a class of linear dynamical systems in which the system and system input matrices, and additionally the input, are uncertain. Uncertainties are assumed to be measurable functions of time who may range in given compact sets.

In this paper, we presented an optimal control approach for robust tracking of linear systems and given largest value of uncertainty parameter for which system under consideration remains stable. The presented work is an extension of results presented in Haihua Tan et al., (2009) for matched uncertainty case. The simulation results clearly show improved robustness for given linear system for different values of uncertainty parameter. The proposed approach is very simple to implement in MATLAB context.

The paper is organized as follows: section 2, discusses the foundation of robust control design, methodology for robust tracking of linear systems for matched uncertainty. In section 3, case study of simple linear system with uncertainty parameter is reviewed, and finally some brief conclusions are given in section 4.

II. ROBUST TRACKING USING STATE FEEDBACK

We first present results obtained in Haihua Tan et al., (2009), that will form the theoretical foundation of our approach.

2.1. Background

For linear systems, the approach considers the following system,

$$\dot{x} = A(p)x + Bu \tag{1}$$

Where $x \in R^n$ are state variables, $u \in R^m$ are control inputs, and $p \in P$ are uncertain parameters. If the uncertainty of $A(p)$ is in the range of B , that is, the uncertainty in $A(p)$ can be written as

$$A(p) - A(p_0) = B\phi(p) \tag{2}$$

for some $\phi(p_0)$, where $p_0 \in p$ is nominal value of p , then we say that the matching condition is satisfied.

2.2. Methodology

An optimal approach is proposed that translates robust control problem to an optimal control problem and if the system under consideration is linear system then optimal control problem will become Linear Quadratic problem. This LQR problem then can be solved by solving an algebraic Riccati equation.

2.3. Robust Tracking using State Feedback for Matched Uncertainty

The linear system is described by

$$\dot{x} = A(p_0)x + Bu + B\phi(p)x \tag{3}$$

$$y = cx \tag{4}$$

Our objective is to design a feedback control law ‘ u ’ under which the output asymptotically tracks the reference signal with no error, that is, $y \rightarrow y_r$ as $t \rightarrow \infty$. In determining the control law, we assume that the following information is available for control:

- (1) State variables x and
- (2) Integrals of error $e = y - y_r$.

To formalise this, let us define new state variables as

$$\dot{q}_1 = e, \dot{q}_2 = q_1, \dots, \dot{q}_d = q_{d-1} \tag{5}$$

The state equation with original and new state variables can be written as,

$$\begin{bmatrix} \dot{x} \\ \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_d \end{bmatrix} = \begin{bmatrix} A(p_0) & 0 & 0 & 0 & x & B \\ C & 0 & 0 & 0 & q_1 & 0 \\ 0 & 1 & 0 & 0 & q_2 & + 0 \times [\phi(p) \ 0 \ 0 \ \dots \ 0] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & q_d & \vdots \\ & & & & & B \\ & & & & & 0 \\ & & & & & -1 \\ & & & & & + 0 \ u + 0 \ y_r \\ & & & & & \vdots \\ & & & & & 0 \end{bmatrix} \begin{bmatrix} x \\ q_1 \\ q_2 \\ \vdots \\ q_d \end{bmatrix}$$

Denote the above equation as

$$\dot{z} = A_z(p_0)z + B_z\phi_z(p)z + B_zu + My_r$$

Thus the overall controlled system structure is shown in figure 1 [Haihua Tan et al., 2009].

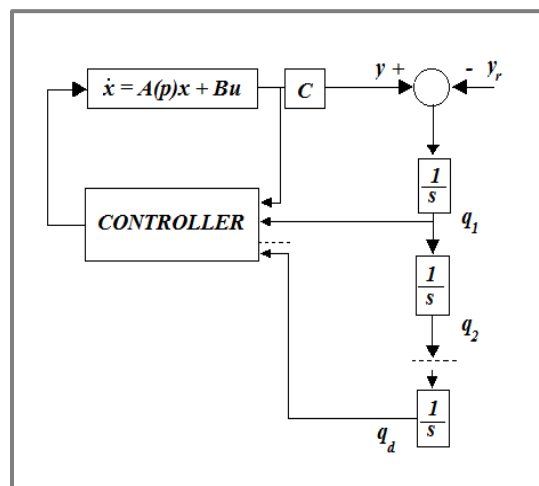


Figure 1: Robust Tracking with State Feedback

For the nominal system,

$$\begin{aligned} \dot{x} &= A(p_0)x + Bu + B\phi(p)x \\ y &= Cx \end{aligned}$$

Find a feedback control law $u = K_z z$ that minimizes the cost function

$$J = \int_0^{\infty} (x^T F_z x + x^T x + u^T u) dt \quad (6)$$

Where F_z is an upper bound on uncertainty $\phi_z^T(p)\phi_z(p)$; that is, for all $p \in P$, $\phi_z^T(p)\phi_z(p) \leq F_z$. The upper bound on the uncertainty $\phi_z(p) = [\phi(p) \ 0 \ 0 \ \dots \ 0]$ is denoted by F_z . We further assume that $(A_z(p_0), B_z)$ is controllable, otherwise, the solution, may not exist.

2.4. Riccati Equation: [Donald E. Kirk, 2004]

Let the system equation given by

$$\dot{x} = a(x, u, t) \quad (7)$$

And if the performance measure is given by

$$J = \int_0^{\infty} g(x, u, t) dt \quad (8)$$

The HJB equation is given by

$$J_t^* + H^* = 0 \quad (9)$$

Where H^* is minimized Hamiltonian function given by,

$$H = g(x, u, t) + (J_x^*(x, t))^T a(x, u, t) \quad (10)$$

From (1) and (6) we get,

$$H = x^T F_z x + x^T x + u^T u + (J_x^*(x, t))^T (A(p_0)x + Bu) \quad (11)$$

$$\frac{\partial H}{\partial u} = 2u + J_x^{*T} B$$

$$\frac{\partial^2 H}{\partial u^2} = 2 \geq 0$$

$$2u = -J_x^{*T} B$$

$$u = -\frac{1}{2} J_x^{*T} B$$

$$u^* = -\frac{1}{2} (J_x^{*T} B)^T \quad (12)$$

Plug in equation (12) in (11) to obtain H^*

$$\begin{aligned} H^* &= x^T F_z x + x^T x + \left(-\frac{1}{2} B^T J_x^*\right)^T \left(-\frac{1}{2} B^T J_x^*\right) \\ &\quad + J_x^{*T} \left(A(p_0)x + B \left(-\frac{1}{2} B^T J_x^*\right) \right) \end{aligned}$$

$$H^* = x^T F_z x + x^T x - \frac{1}{4} J_x^{*T} B B^T J_x^* + J_x^* A(p_0)x$$

Thus from (9)

$$J_t^* + x^T F_z x + x^T x - \frac{1}{4} J_x^{*T} B B^T J_x^* + J_x^* A(p_0)x = 0$$

The one of the solution of the form is given by,

$$J^*(x(t), t) = x^T S_z x \quad (13)$$

Thus,

$$J_x^* = 2S_z x \quad \text{and} \quad J_t^* = 0$$

Substituting these values in HJB equation to obtain Riccati equation,

$$0 = 0 + x^T F_z x + x^T x - \frac{1}{4} (2S_z x)^T B B^T (2S_z x) + (2S_z x)^T A(p_0)x$$

But in new state variable form $B = B_z$,

$$x^T F_z x + x^T x - x^T S_z B_z B_z^T S_z x + 2x^T S_z A_z(p_0)x = 0$$

But $S_z A_z(p_0)$ can be written as,

$$S_z A_z(p_0) = \frac{1}{2} [S_z A_z(p_0) + A_z^T(p_0) S_z]$$

$$x^T F_z x + x^T x - x^T S_z B_z B_z^T S_z x + x^T [S_z A_z(p_0) + A_z^T(p_0) S_z] x = 0$$

$$x^T [F_z + I - S_z B_z B_z^T S_z + S_z A_z(p_0) + A_z^T(p_0) S_z] x = 0$$

Since x can't be zero, thus

$$F_z + I - S_z B_z B_z^T S_z + S_z A_z(p_0) + A_z^T(p_0) S_z = 0 \quad (14)$$

This is known as Riccati equation.

To solve the LQR problem, we first solve the algebraic Riccati equation for S_z . Then the solution to the LQR problem is given by

$$u = Kz = -B_z^T S_z z \quad (15)$$

So the control law is given by (17),

We now show that the system,

$$\dot{z} = A_z(p)z + B_z K_z z$$

$$= A_z(p_0)z + B_z K_z z + B_z \phi_z(p)z \quad (16)$$

is asymptotically stable for all $p \in P$. Now, let us consider the following Lyapunov function candidate:

$$V(z) = z^T S_z z$$

Clearly,

$$V(z) > 0, \quad z \neq 0$$

$$V(z) = 0, \quad z = 0$$

To show $\dot{V}(z) < 0$ for all $z \neq 0$, we have

$$\dot{V}_z = \dot{z}^T S_z z + z^T S_z \dot{z}$$

$$\dot{V}_z = (A_z(p_0)z + B_z K_z z + B_z \phi_z(p)z)^T S_z z + z^T S_z$$

$$(A_z(p_0)z + B_z K_z z + B_z \phi_z(p)z)$$

$$\dot{V}_z = z^T (A_z^T(p_0) S_z + S_z A_z(p_0) + 2S_z B_z K_z +$$

$$S_z B_z B_z^T S_z + \phi_z^T(p)\phi_z(p)) z$$

$$\dot{V}_z = z^T (A_z^T(p_0) S_z + S_z A_z(p_0) - 2S_z B_z B_z^T S_z + S_z B_z B_z^T S_z + \phi_z^T(p)\phi_z(p)) z$$

Because

$$\phi_z^T(p) B_z^T S_z + S_z B_z \phi_z(p) \leq S_z B_z B_z^T S_z + \phi_z^T(p)\phi_z(p)$$

We have

$$\dot{V}(z) \leq z^T (A_z^T(p_0) S_z + S_z A_z(p_0) - 2S_z B_z B_z^T S_z$$

$$+ S_z B_z B_z^T S_z + \phi_z^T(p)\phi_z(p)) z$$

$$\dot{V}(z) = z^T (A_z^T(p_0) S_z + S_z A_z(p_0) - S_z B_z B_z^T S_z + \phi_z^T(p)\phi_z(p)) z$$

$$\dot{V}(z) = z^T (A_z^T(p_0) S_z + S_z A_z(p_0) - S_z B_z B_z^T S_z + F_z) z$$

In other words,

$$\dot{V}(z) < 0, \quad z \neq 0.$$

$$\dot{V}(z) = 0, \quad z = 0.$$

By the Lyapunov stability Theorem, the controlled system is asymptotically stable and $z \rightarrow 0$ as $t \rightarrow \infty$.

Let us now consider the extended system,

$$\dot{z} = A_z(p_0)z + B_z \phi_z(p)z + B_z u + My_r$$

Taking the d^{th} derivative on both sides, we have

$$z^{(d+1)} = A_z(p)z^d + B_z K_z z^d.$$

This equation has the same form as

$$\dot{z} = A_z(p)z + B_z K_z z.$$

Therefore,

$$z^d \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

In particular,

$$q_d^d = y - y_r \rightarrow 0 \text{ as } t \rightarrow \infty.$$

III. CASE STUDY

Example 1 [Haihua Tan et al., 2009]:

Let us consider the following second-order system,

$$\dot{x} = \begin{bmatrix} 0 & 5 \\ 5+p & p \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0]x$$

where $p \in [0,10]$ is the uncertainty. We would like to design a state feedback control law to ensure that the output y is tracking a reference signal $y_r = 1$ for all $p \in [0,10]$. We first construct the augmented system as follows

$$\dot{z} = A_z(p)z + B_z u + M y_r$$

where

$$z(t) = \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} \quad A_z(p) = \begin{bmatrix} A_z(p) & 0 \\ C & 0 \end{bmatrix}$$

$$A_z(p) = \begin{bmatrix} 0 & 5 & 0 \\ 5+p & p & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad B_z = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and $M = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

To solve this problem, let us take $P_0 = 1$ and verify that $(A_z(p_0), B_z)$ is controllable. We then calculate F_z as follows.

$$\Delta A_z = \begin{bmatrix} \Delta A(p) & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ p & p & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Delta A_z(p) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [p \ p \ 0] = B_z \phi_z(p)$$

$$\phi_z^T(p) \phi_z(p) = \begin{bmatrix} p \\ p \\ 0 \end{bmatrix} [p \ p \ 0] = \begin{bmatrix} p^2 & p^2 & 0 \\ p^2 & p^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\leq \begin{bmatrix} 100 & 100 & 0 \\ 100 & 100 & 0 \\ 0 & 0 & 0 \end{bmatrix} = F_z$$

Hence, we solve the following algebraic Riccati equation

$$A_z^T(p_0)S_z + S_z A_z(p_0) - S_z B_z B_z^T S_z + F_z + I = 0$$

To obtain

$$S_z = \begin{bmatrix} 17.3593 & 16.5120 & 3.2626 \\ 16.5120 & 16.3132 & 1 \\ 3.2626 & 1 & 11.5120 \end{bmatrix}$$

Finally the solution to the robust tracking using state feedback exists and is given by following state feedback matrix.

$$K_z = -B_z^T S_z = - [16.5120 \ 16.3132 \ 1]$$

To see the performance of the closed-loop system, we conduct simulations, whose results are shown in Figure2-5, for $p = 1$ and $p = 11$ respectively.

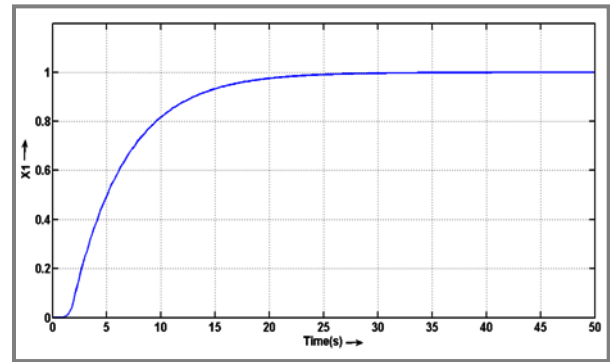


Figure 2: Simulation Result for X1 of Example 1 for p=1

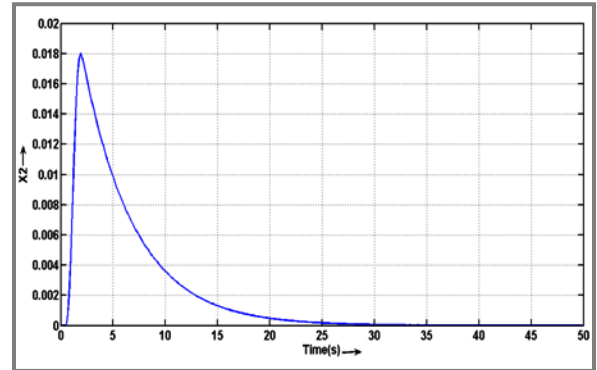


Figure 3: Simulation Result for X2 of Example 1 for p=1

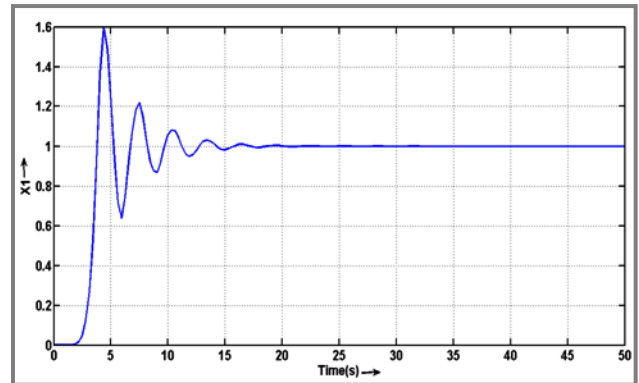


Figure 4: Simulation Result for X1 of Example 1 for p=11

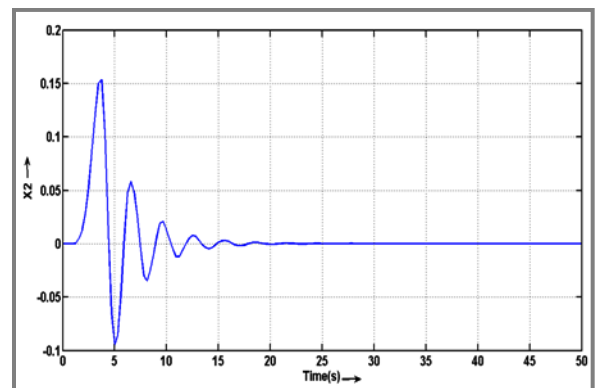


Figure 5: Simulation Result for X2 of Example 1 for p=11

Here $X1$ and $X2$ are the states of system. From the simulation results of figure: 2-3, we see that the system response is not fast. This is because the poles of the controlled system are not far away from the imaginary axis, that is, the stability margin is not large. Also from figure: 4-5,

for $p=11$ it can be seen that response is under damped response. This implies that as p increases system moves towards instability. Thus we can conclude that system is less robust. If we want a large stability margin and fast response, we may want to place the poles of the controlled system at least λ distance away from the imaginary axis. In other words, we want to solve the following problem [Haihua Tan et al., 2009].

3.1. Robust Tracking Problem with Guaranteed Stability Margin

For an arbitrary positive real number λ , find a feedback control law $u = K_z Z$ such that the controlled system

$$\begin{aligned} \dot{x} &= A(p_0)x + Bu + B\phi(p)x \\ &= A(p_0)x + BK_z z + B\phi(p)x \\ y &= Cx \end{aligned}$$

has all its poles on the left of $-\lambda$ and $y \rightarrow y_r$ for all possible $p \in P$. The solution to the above problem is given as follows.

Assume that the matching condition is satisfied and there exist a nominal value $p_0 \in P$ such that $(A_z(p_0), B_z)$ is controllable. Then the solution to the robust tracking problem with guaranteed stability margin exists and is given by $u = K_z Z = -B_z^T S_z Z$, where S_z is the solution to the algebraic Riccati equation

$$\begin{aligned} (A_z(p_0) + \lambda I)^T S_z + S_z (A_z(p_0) + \lambda I) - \\ S_z B_z B_z^T S_z + F_z + I = 0 \end{aligned}$$

Since $(A_z(p_0), B_z)$ and hence $(A_z(p_0) + \lambda I, B_z)$ is controllable (for all positive real λ) the solution to the algebraic Riccati equation exists. We know its solution $u = K_z Z = -B_z^T S_z Z$ has the following property: the system

$$\begin{aligned} \dot{z} &= A_z(p) + B_z K_z z + \lambda z \\ &= A_z(p_0)z + B_z K_z z + B_z \phi_z(p)z + \lambda z \end{aligned}$$

is asymptotically stable and $y \rightarrow y_r$ for all possible $p \in P$. Denote real part of complex numbers s by $Re(s)$ and the determinant of matrix A by $|A|$.

Then we have

$$\begin{aligned} (\forall p \in P)(\forall s, Re(s) \geq 0) |sI - A_z(p_0) - \lambda I - B_z K_z - B_z \phi_z(p)| \neq 0 \\ \Rightarrow (\forall p \in P)(\forall s, Re(s) \geq 0) (s - \lambda) |sI - A_z(p_0) - B_z K_z - B_z \phi_z(p)| \neq 0 \end{aligned}$$

Let $s' = s - \lambda$ then

$$\begin{aligned} Re(s) = Re(s' + \lambda) = Re(s') + \lambda \geq 0 \\ \Leftrightarrow Re(s') \geq -\lambda. \end{aligned}$$

Therefore,

$$(\forall p \in P)(\forall s', Re(s') \geq -\lambda) |s'I - A_z(p_0) - B_z K_z - B_z \phi_z(p)| \neq 0$$

which implies that the controlled system has all its poles on the left of $-\lambda$. Furthermore, it is clear that $y \rightarrow y_r$ for all possible $p \in P$.

Example 2 [Haihua Tan et al., 2009]:

Consider the system discussed in example 1, we would like to design a state feedback control to ensure that the output y is tracking a reference signal $y_r = 1$ and has all its poles on the left of -2 (that is, $\lambda=2$) for all $p \in [0,10]$. The augmented system is same as in Example 1, but the algebraic Riccati equation to be solved is different:

$$(A_z(p_0) + \lambda I)^T S_z + S_z (A_z(p_0) + \lambda I) - S_z B_z B_z^T S_z + F_z + I = 0$$

The solution is given by

$$S_z = \begin{bmatrix} 79.1179 & 34.7323 & 220.7693 \\ 34.7323 & 23.2679 & 57.2894 \\ 220.7693 & 57.2894 & 820.27 \end{bmatrix}$$

And the feedback matrix is given by

$$K_z = -B_z^T S_z = -[34.7323 \quad 23.2679 \quad 57.2894]$$

The simulation results are shown in figure 6-9.

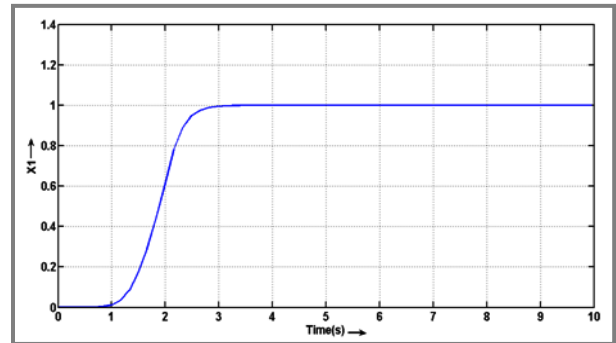


Figure 6: Simulation Result for X1 of Example 2 for p=1

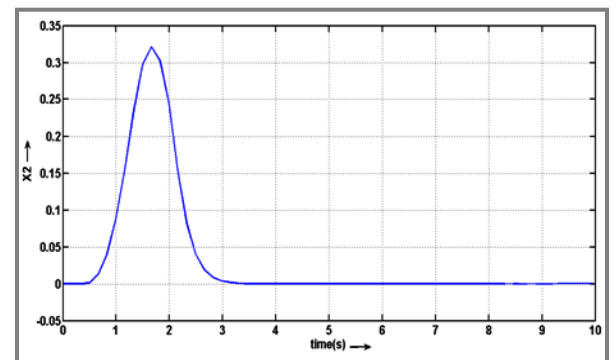


Figure 7: Simulation Result for X2 of Example 2 for p=1

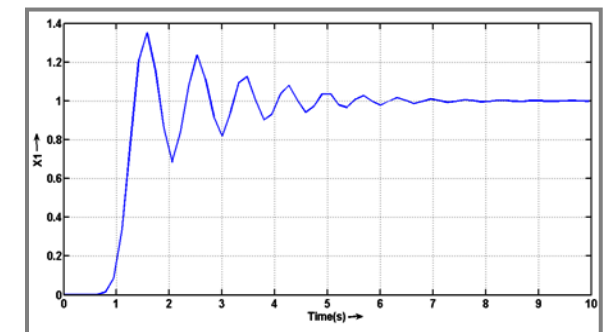


Figure 8: Simulation Result for X1 of Example 2 for p=17

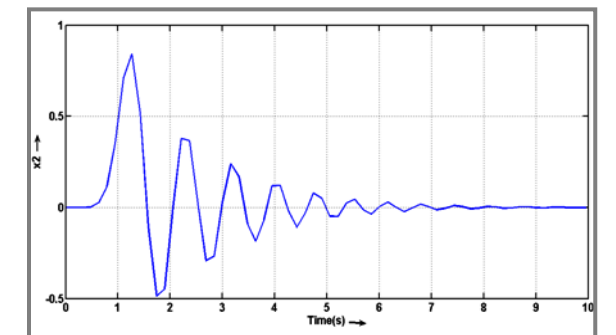


Figure 9: Simulation Result for X2 of Example 2 for p=17

From figure 2-3 and figure 6-7 it can be seen that the tracking response of system in example 1 is much slower than system defined in example 2. Also from figure 4-5 and figure 8-9 it can be seen that system defined in example 2 is robust than system defined in example 1.

IV. CONCLUSION

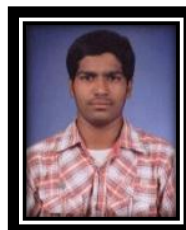
In this paper, we explored the robust tracking problem of a linear system for matched uncertainty and for different values of p its robustness property investigated. To represent errors and their integrals new state variables were introduced. For this state feedback control approach is proposed. A state feedback matrix is obtained by solving an algebraic Riccati equation. The controller design was based on an optimal control approach. The closed-loop system was proved to be robustly stable and track the desired signals.

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