A Nonlinear Model Reference Adaptive Control for a Universal Motor using Backstepping Approach

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***Abstract*—**Our aim is to design a nonlinear controller for the given Universal motor. In order to attain high performance speed tracking, the adaptive backstepping control technique is applied. A linear reference model is designed to achieve the speed tracking of the motor. To regulate the dynamics of the motor, the proposed controller is designed to track the linear reference model. Considering the parameter uncertainties, adaptive control method is used. The effectiveness of the controller is verified using the simulation results. The simulation results show that the motor speed response is highly robust despite the uncertainties and load torque variation and the motor also successfully tracks the specified reference model.

***Keywords*—**Model Reference Adaptive Backstepping Control; Speed Control; Universal Motor (UM).

***Abbreviations*—**Back Electromotive Force (BEMF); Model Reference Adaptive Backstepping (MRAB); Universal Motor (UM).

I. INTRODUCTION

THE popularity and application of the Universal Motor (UM) has grown rapidly over the years, specifically in domestic use. Home appliances such as vacuum cleaners, mixers, drills, saws, etc. The UM is a very powerful motor compared to its size. It has many advantages like high starting and running torque, inexpensive to manufacture, variable speed that can be regulated, etc. Despite many advantages, it has shorter span of life compared to other motors and requires high maintenance [Gregor Papa, 2003].

Feedback linearization control [Alberto Isidori, 1995] has been thoroughly studied in the last 20 years, by which the original nonlinear model can be transformed into a linear model through proper coordinate transformation. Thus, almost all the well-developed linear control techniques might be applied. But when parameter uncertainties and unknown disturbance are taken into account, this approach may not be applicable because it is based on the exact cancellation of the nonlinearity.

Backstepping control is a newly developed technique to the control of uncertain nonlinear systems, particularly those systems that do not satisfy matching conditions [Krstic et al., 1995]. The most engaging point of it is to use the virtual control variable to make the original high order system to be simple enough thus the final control outputs can be derived step by step through suitable Lyapunov functions.

A nonlinear torque controller for an induction motor was designed based on adaptive backstepping approach, in which over parameterization may occur [Shei & Shyu, 1999]. In that paper, there are only two uncertainties, however in the adaptation laws, there are totally 5 uncertainty parameters. Thus the problem of over parameterization happened. A speed controller was based on backstepping was developed for induction motors, but there was a possible singularity problem [Tan & Chang, 1999].

A reference model is used to give the desired transient performance in order to get the error model and thus make the derivation of the control scheme easily. Finally nonlinear adaptive speed controller based on nonlinear adaptive backstepping control technique is derived step by step. It has no singularity and over parameterization [Jianguo Zhou & Youyi Wang, 2005]. The resulted control scheme can track the reference signal quite well under parameter uncertainties and load torque disturbance.

This paper proposes a novel strategy for improving steady performance in a Universal motor drive using the model reference adaptive backstepping (MRAB) control [Hai

ISSN: 2347-6087

Lin, 2009]. Since the MRAB method has the robustness properties of them is matched uncertainties and disturbance, the drive with MRAB controller achieves the quickly and steady dynamic performance. The simulation results show that the proposed method has a much better speed tracking performance accurately while keeping a good dynamic performance. Mismatched uncertainties and disturbance, the drive with MRAB controller achieve the quickly and steady dynamic.

II. MODEL OF UNIVERSAL MOTOR

The differential equations of the universal motor have been derived from its equivalent model. The universal motor is basically a series wound machine with field winding connected in series with the rotor winding [Pankaj Sahu, 2013].

The system equations can be written as follows,

$$v(t) = (R_f + R_a)i(t) + (L_f + L_a)\frac{di}{dt} + e(t) \tag{1}$$

Where,

$$e(t) = ki(t)\omega_m(t) \tag{2}$$

This is the electromechanical part of the modeling. R_a is the armature resistance, R_f is the field resistance. L_a and L_f are the armature and field inductances respectively. e(t) is the back emf and k is the back emf constant.

The mechanical part of the modeling is explained below.

$$T_e = T_L + B\omega_m + J\frac{d\omega_m}{dt} \tag{3}$$

Where,

$$T_e = ki^2(t) \tag{4}$$

T_e is the electromagnetic torque produced by the motor, T_L is the load torque, B is the friction coefficient and J is the motor inertia.

III. MODEL REFERENCE ADAPTIVE BACKSTEPPING CONTROL

In the traditional current control scheme of Universal motors the voltage equation (1) of the Universal motors is rewritten by

$$u_s = (R_f + R_a)i_s + (L_f + L_a)\dot{i}_s + E \tag{5}$$

Where u_s and i_s are the control voltage and armature current respectively.

The electromagnetic torque (3) for two phases combined can be expressed as

$$T_e = k_T i^2{}_s \tag{6}$$

Where $k_e = k_t$, which is the moment coefficient of the motor and assuming $R_a=R_f=R$ and $L_a=L_f=L$.

Considering the equations (4)-(6), the mathematical model of the motor can be written as

$$\begin{cases} \dot{i}_s = \dfrac{(u_s - 2Ri_s - E)}{2L} \\ \dot{w} = \dfrac{(-B\omega + T_e - T_L)}{J} \end{cases} \tag{7}$$

While developing the adaptation laws, it is assumed that stator resistance, the friction coefficient and the load torque are the unknown but constant parameters in control system, the following variables are defined

$$\begin{cases} R = R_N + \Delta R \\ B = B_N + \Delta B \\ T_L = T_{LN} + \Delta T_L \end{cases} \tag{8}$$

Where the nominal values of the stator resistance, the friction coefficient and the load torque are R_N, B_N and T_{LN} and ΔR, ΔB and ΔT_L are the errors between the real values and the nominal values of the stator resistance, the friction coefficient and the load torque.

Considering the uncertainties in (8), the system can be described as

$$\dot{x} = f(x) + \Delta f(x) + g(x)u_s \tag{9}$$

Where,

$$x = \begin{pmatrix} i_s \\ \omega \end{pmatrix}, f(x) = \begin{pmatrix} (-2R_N i_s - k_T\omega)/2L \\ (-B_N\omega + k_T i * i - T_{LN})/J \end{pmatrix},$$

$$\Delta f(x) = \begin{pmatrix} -\Delta R_N i_s / L \\ (-\Delta B\omega - \Delta T_L)/J \end{pmatrix}, g(x) = \begin{pmatrix} 1/2L \\ 0 \end{pmatrix}$$

The system controller objective is defined as

$$\begin{cases} z_1 = h(x) = \omega \\ z_2 = L_f h(x) \end{cases} \tag{10}$$

Where the notation is used for "Lie Derivative" of a state function h(x): $R^n \rightarrow R$ along the direction of a field f(x) [Hassan K Khalil, 2002].

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x} f_i(x) \tag{11}$$

And similarly $L_f^i h(x) = L_f(L_f^{i-1}h(x))$.

From (9)-(10), the following dynamic can be obtained for the system shown in (7)

$$\begin{cases} \dot{z}_1 = L_f h(x) + \omega\theta_1 + \theta_2 \\ \dot{z}_2 = L_f^2 h(x) + L_g L_f h(x)u_s + L_{\Delta f} L_f h(x) \end{cases} \tag{12}$$

The uncertain parameters are assumed to be

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \Delta B / J \\ \Delta T_L / J \\ \Delta R / L \end{bmatrix} \tag{13}$$

From (7)-(13), the state space model of the motor is given as

$$\begin{cases} \dot{z}_1 = L_f h(x) + \omega\theta_1 + \theta_2 \\ \dot{z}_2 = L_f^2 h(x) + \bar{u}_s - B_N\omega\theta_1/J - B_N\theta_2/J + 2k_T i^2{}_s\theta_3/J \end{cases} \tag{14}$$

Where $\bar{u}_s = L_g L_f h(x) u_s$

The above system is a nonlinear system, to assign the desired output response of the system (14), a linear reference model is designed as

$$\begin{cases} \dot{z}_{m1} = z_{m2} \\ \dot{z}_{m2} = -k_{m1}z_{m1} - k_{m2}z_{m2} + k_{m1}\omega * \end{cases} \quad (15)$$

Where z_{m1} and z_{m2} are the reference states,k_{m1} and k_{m2} are the desired gains, ω* is the reference speed command.

Furthermore, the tracking error between the dynamic system (14) and the reference model (15) as

$$e_z = \begin{pmatrix} e_{z1} \\ e_{z2} \end{pmatrix} = \begin{pmatrix} z_1 - z_{m1} \\ z_2 - z_{m2} \end{pmatrix} \quad (16)$$

According to (14)-(16), the state equations in terms of error variables are given by,

$$\begin{aligned} \dot{e}_{z1} &= e_{z2} + (e_{z1} + z_{m1})\theta_1 + \theta_2 \\ \dot{e}_{z2} &= L^2{}_f h(x) + \tilde{u}_s - B_N \omega \frac{\theta_1}{J} - B_N \frac{\theta_2}{J} + 2k_T i^2{}_s \frac{\theta_3}{J} \end{aligned} \quad (17)$$

Where $\tilde{u}_s = \bar{u}_s + k_{m1}z_{m1} + k_{m2}z_{m2} - k_{m1}\omega^*$

We know that parameter uncertainties exist in the system, considering them in the designed controller, we define

$$\begin{aligned} \tilde{\theta}_1 &= \theta_1 - \hat{\theta}_1 \\ \tilde{\theta}_2 &= \theta_2 - \hat{\theta}_2 \\ \tilde{\theta}_3 &= \theta_3 - \hat{\theta}_3 \end{aligned} \quad (18)$$

Where $\hat{\theta}_1, \hat{\theta}_2 and \hat{\theta}_3$ are the estimations of $\theta_1, \theta_2 and \theta_3$, $\tilde{\theta}_1, \tilde{\theta}_2 and \tilde{\theta}_3$ are the estimations errors.

Then, define the new error variables as

$$\bar{e}_z = \begin{pmatrix} \bar{e}_{z1} \\ \bar{e}_{z2} \end{pmatrix} = \begin{pmatrix} e_{z1} \\ e_{z2} - \alpha \end{pmatrix} \quad (19)$$

Where α is a stabilizing function for e_{z2}. That is, e_{z2} canstably converge to α, which can be considered as the control for the first equation of (17). The stabilizing control function is defined as

$$\alpha = -k_1 e_{z1} - e_{z1}\hat{\theta}_1 - z_{m1}\hat{\theta}_1 - \hat{\theta}_2 \quad (20)$$

Where k_1 is a positive constant.

Taking the derivative of (19) with respect to time and then substituting (17) and (20) into the derivative, the new error dynamics is given by

$$\begin{aligned} \dot{\bar{e}}_{z1} &= \bar{e}_{z2} - k_1\bar{e}_{z1} + \bar{e}_{z1}\tilde{\theta}_1 + z_{m1}\tilde{\theta}_1 + \tilde{\theta}_2 \\ \dot{\bar{e}}_{z2} &= L^2{}_f h(x) + \tilde{u}_s - B_N\omega(\tilde{\theta}_1 + \hat{\theta}_1)/J - B_N(\tilde{\theta}_2 + \hat{\theta}_2)/J \\ &\quad + 2k_T i^2{}_s(\tilde{\theta}_3 + \hat{\theta}_3)/J + (k_1 + \hat{\theta}_1)\dot{\bar{e}}_{z1} \\ &\quad + (\bar{e}_{z1} + z_{m1})\dot{\hat{\theta}}_1 + \dot{z}_{m1}\hat{\theta}_1 + \dot{\hat{\theta}}_2 \end{aligned} \quad (21)$$

To design the backstepping control system, the parameter uncertainties are assumed to be bounded and define the following Lyapunov function,

$$V = \frac{1}{2}\bar{e}_{z1}{}^2 + \frac{1}{2}\bar{e}_{z2}{}^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^2 + \frac{1}{2\gamma_3}\tilde{\theta}_3^2 \quad (22)$$

Where, γ_1, γ_2 and γ_3 are adaptation gains.

Using (20) and (21), the derivative of the Lyapunov function (22) can be derived as

$$\begin{aligned} \dot{V} &= \bar{e}_{z1}\dot{\bar{e}}_{z1} + \bar{e}_{z2}\dot{\bar{e}}_{z2} + \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\tilde{\theta}}_1 + \frac{1}{\gamma_2}\tilde{\theta}_2\dot{\tilde{\theta}}_2 + \frac{1}{\gamma_3}\tilde{\theta}_3\dot{\tilde{\theta}}_3 \\ \dot{V} &= \bar{e}_{z2}(\bar{e}_{z1} + L_f^2 h_1 + \bar{u}_s - B_N\omega\hat{\theta}_1/J - B_N\hat{\theta}_2/J + \\ &\quad 2k_T i^2{}_s\hat{\theta}_3/J + k_1\bar{e}_{z2} - k_1^2\bar{e}_{z1} + \hat{\theta}_1\bar{e}_{z2} - k_1\hat{\theta}_1\bar{e}_{z1} + \dot{\hat{\theta}}_1\bar{e}_{z1} + \\ &\quad \dot{\hat{\theta}}_1 z_{m1} + \hat{\theta}_1\dot{z}_{m1} + \dot{\hat{\theta}}_2) - k_1 e_{z1}^2 + \tilde{\theta}_1(\bar{e}_{z1}^2 - B_N\omega\bar{e}_{z2}/J + \\ &\quad k_1\bar{e}_{z1}\bar{e}_{z2} + k_1 z_{m1}\bar{e}_{z2} + \hat{\theta}_1\bar{e}_{z1}\bar{e}_{z2} + \hat{\theta}_1 z_{m1}\bar{e}_{z2} + \bar{e}_{z1}z_{m1} \\ &\quad - \dot{\hat{\theta}}_1/\gamma_1) + \tilde{\theta}_2(\bar{e}_{z1} - B_N\bar{e}_{z2}/J + k_1\bar{e}_{z2} + \hat{\theta}_1\bar{e}_{z2} - \\ &\quad \dot{\hat{\theta}}_2/\gamma_2) + \tilde{\theta}_3(2k_T i^2{}_s\bar{e}_{z2}/J - \dot{\hat{\theta}}_3/\gamma_3) \end{aligned} \quad (23)$$

To ensure the global asymptotical stability, we must satisfied the following inequality

$$\dot{V} \le 0 \quad (24)$$

According to (23) and (24), a backstepping control can be designed as below,

$$\begin{aligned} \tilde{u}_s &= -k_2\bar{e}_{z2} - \bar{e}_{z1} - L_f^2 h_1 + B_N\omega\hat{\theta}_1/J + B_N\hat{\theta}_2/J - \\ &\quad 2k_T i^2{}_s\hat{\theta}_3/J - k_1\bar{e}_{z2} + k_1^2\bar{e}_{z1} - \hat{\theta}_1\bar{e}_{z2} + k_1\hat{\theta}_1\bar{e}_{z1} - \dot{\hat{\theta}}_1\bar{e}_{z1} - \\ &\quad \dot{\hat{\theta}}_1 z_{m1} - \hat{\theta}_1\dot{z}_{m1} - \dot{\hat{\theta}}_2 \end{aligned} \quad (25)$$

Parameter adaptation laws for $\dot{\hat{\theta}}_1, \dot{\hat{\theta}}_2 and \dot{\hat{\theta}}_3$ are designed as

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \gamma_1(\bar{e}_{z1}^2 - B_N\omega\bar{e}_{z2}/J + k_1\bar{e}_{z1}\bar{e}_{z2} + k_1 z_{m1}\bar{e}_{z2} + \\ &\quad \hat{\theta}_1\bar{e}_{z1}\bar{e}_{z2} + \hat{\theta}_1 z_{m1}\bar{e}_{z2} + \bar{e}_{z1}z_{m1}) \\ \dot{\hat{\theta}}_2 &= \gamma_2(\bar{e}_{z1} - B_N\bar{e}_{z2}/J + k_1\bar{e}_{z2} + \hat{\theta}_1\bar{e}_{z2}) \\ \dot{\hat{\theta}}_3 &= 2\gamma_3 k_T i^2{}_s\bar{e}_{z2}/J \end{aligned} \quad (26)$$

IV. SIMULATION RESULTS

To investigate the effectiveness of the proposed adaptive backstepping control system for the motor, a simulation model is implemented in Matlab.

The system drive is started at no load and increased to 5N · m at t= 1s. The speed reference changed from 0 to 500rad/ s at t=0s. Then, at t=5s, the reference speed is increased to 1000rad/ s. The stator resistance and viscous coefficient of the motor are given with the nominal values, i.e., $R = R_n$ and $B = B_n$. In order to verify the robustness to the change of the system parameters, the stator resistance and viscous coefficient are increased to $R = 2R_n$ and $B = 2B_n$.

Considering the dynamic response of the reference model and system model, the parameters of the reference model (15) are chosen to be $k_{ml} = 155$ and $k_{m2} = 30$, the parameters of the proposed controller (25) are chosen as $k_l = 1200$ and $k_2 = 120$, and the adaptive gains in (26) are given as γ_1= 0.000002, γ_2= 0.000001 and γ_3 = 0.000001 [Hai Lin, 2009].

To demonstrate the dynamic performance of the universal motor with the proposed control scheme, we design a simulink model in matlab with the help of the equations presented above. The fig (2)-(6) show the simulation results of the given motor.

Initially we apply desired step input having a magnitude of 500 units, which is the speed command. After 5s we add another step input having the same magnitude. The reference model designed in the previous section eventually settles at its equilibrium point i.e. ω^* (speed command), which means that the reference signal converges at the input command. Hence the figure consists of the response of the reference model and the actual speed response of the motor. The aim of the motor is to track the given reference signal. We see and conclude that the performance of the controller is fairly good as it is successfully tracking the command.

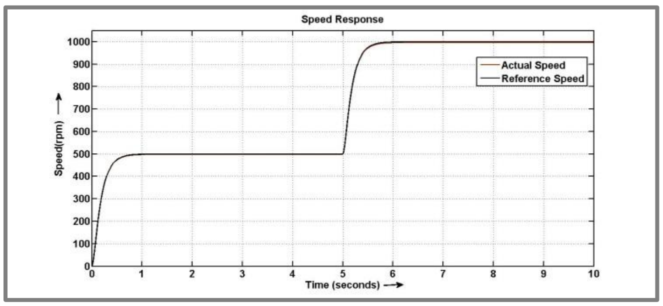

Figure 2: Motor Speed v/s Time

Figure (3) shows the speed error i.e. figure shows the error in speed of the motor i.e. difference between the actual speed and the reference signal. We see that the speed error is around 2 rpm till 5s and 3 rpm after adding another step command.

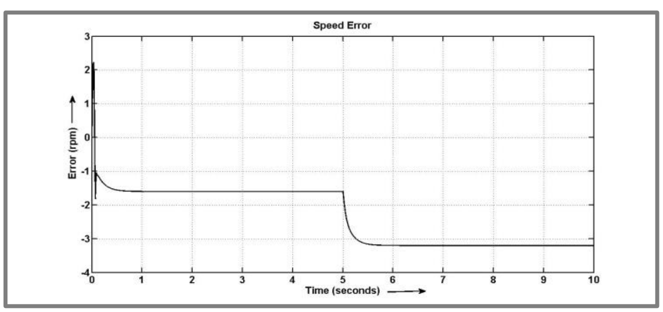

Figure 4: Estimated Parameter 1 v/s Time

Our second estimated parameter is load torque T_L and the parameter has converged to a stable value. Even after disturbance at 5s it maintains its steady value.

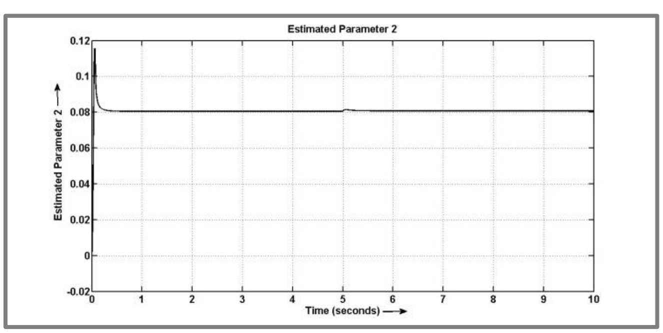

Figure 5: Estimated Parameter 2 v/s Time

Our third estimated parameter is resistance R and the parameter has converged to a stable value. Even after disturbance at 5s it maintains its steady value.

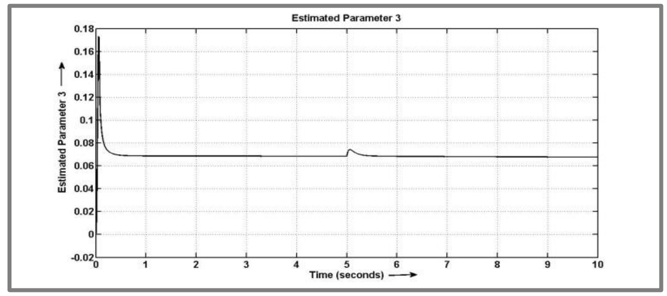

Figure 6: Estimated Parameter 3 v/s Time

Figure 4, 5, 6 shows the trajectory of estimated parameter 1 (B), estimated parameter 2 (T_L) and estimated parameter 3 (R).

V. Conclusion

A nonlinear speed control scheme for universal motor has been proposed in the paper. An adaptive backstepping controller is designed to regulate the speed and achieve quickly and accurately speed tracking performance by the definition of a linear reference model. The simulation results show the improvements with regard to the robustness of parameter uncertainties and load disturbances in the proposed scheme. We see that the controller performance has been very good even though the load torque is varying and the parameters of the system are unknown. Moreover, the torque ripple during the commutation is one of the most critical problems to be solved by future activities.

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