Variance of Time to Recruitment in a Two Graded Manpower System using Order Statistics for Attrition

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***Abstract*—**An organization with two grades subjected to loss of man power due to the policy decisions taken by the organization is considered in this paper As the exit of personal is unpredictable, a new recruitment policy involving two thresholds for each grade one is optional and the other is mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach three mathematical models are constructed using an appropriate univariate policy of recruitment. Performance measures namely mean and variance of the time to recruitment is obtained for the models when (i) the loss of man-hours form an order statistics (ii) the inter-decision time forms a sequence of independent and identically distributed exponential random variables (iii) the optional and the mandatory thresholds follow different distributions. The analytical results are substantiated by numerical illustrations and the influence of nodal parameters on the performance measures is also analyzed.

***Keywords*—**Man Power Planning; Mean and Variance of the Time for Recruitment; Order Statistics; Shock Model; Univariate Recruitment Policy.

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I. INTRODUCTION

RANDOM depletion of manpower occurs in any marketing organization due to the attrition of personnel when the management takes policy decisions regarding pay, perquisites and targets. This attrition will adversely affect the smooth functioning of the organization in due course of time when the loss of man power is not compensated by recruitment. Frequent recruitment is not advisable as it involves more cost. In view of this situation organization should frame a suitable recruitment policy to plan for recruitment. In this context, for a two -graded organization three mathematical models are constructed in this paper using a univariate recruitment policy based on shock model approach.

As the loss of man-hours is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. Esary et al., (1973) have stated a replacement policy for a device, which is exposed to shocks. A number of models can be seen from Grinold & Marshall (1977), Bartholomew & Forbes (1979). The problem of finding the time to recruitment is studied for a single grade and multi grade system by several authors under different conditions. Recently Muthaiyan et al., (2009) have obtained system characteristic for a single grade man-power system when the inter-decision times form an order statistics.

For a single graded system, Esther Clara (2012), has considered a recruitment policy involving two thresholds for the loss of manpower in the organization in which one is optional and the other is mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter-decision time are independent and identically distributed random variables or the inter-decision time are exchangeable and constantly correlated exponential random variables. Srinivasan and Vasudevan (2011A-D) have extended the results of Esther Clara (2012) for a two-grade system according as the thresholds are exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables.

Sridharan et al., (2012A-C & 2013A-D) have extended the results of Muthaiyan et al., (2009) for a two-grade system involving two thresholds by assuming different distributions for thresholds. In the above cited research works of Muthaiyan et al., (2009) and Sridharan et al., (2012A-C & 2013A-D) it is assumed that the inter-decision time form an order statistics and loss of man-hours forms a sequence of independent and identically distributed exponential random variables.

The objective of the present paper is to obtain the mean and variance of the time to recruitment for a two grade system using a univariate recruitment policy assuming that (i) the inter–decision time forms an sequence of independent and identically distributed exponential random variables (ii) the loss of man-hours form an order statistics and (iii) the thresholds for the loss of man-hours in each grade follow different distributions.

II. Model Description and Analysis of Model–I

Consider an organization taking decisions at random epoch in (0, ∞) and at every decision epoch a random number of persons quit the organization. There is an associated loss of man-hours if a person quits. It is assumed that the loss of man-hours are linear and cumulative. Let X_i be the loss of man-hours due to the i^{th} decision epoch, i=1,2,3 Let $X_{(1)}, X_{(2)}, X_{(3)}, \dots, X_{(k)}$ be the order statistics selected from the sample $X_1, X_2, X_3, \dots, X_k$ with respective density functions $g_{x(1)}(.), g_{x(2)}(.), \dots, g_{x(k)}(.)$. Let the inter-decision times are independent and identically distributed with cumulative distribution function F(.), probability density function f(.). Let Y_1, Y_2 (Z_1, Z_2) be the random variables denoting optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with cumulative distribution function H(.), probability density function is h(.). It is assumed that $Y_1<Z_1$ and $Y_2<Z_2$. Write Y=Max (Y_1, Y_2) and Z=Max (Z_1, Z_2) where Y(Z) is the optional (mandatory) threshold for the loss of man-hours in the organization. The loss of man-hours, optional and the mandatory thresholds are statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function L(.), probability density function l (.), mean E(T) and variance V(T). Let $F_k(.)$ be the k fold convolution of F(.). Let $l^*(.)$ and $f^*(.)$, be the Laplace transform of l(.) and f(.), respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in (0, t]. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y. The univariate recruitment policy employed in this paper is as follows: If the total loss of man-hours exceeds the optional threshold Y, the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z, the recruitment is necessary.

Main Results

We note that

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i \le Y\right) + p \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i > Y\right) \times P\left(\sum_{i=1}^{k} X_i < Z\right) \quad (1)$$

For r=1, 2, 3…k the probability density function of $X_{(r)}$ is given by

$$g_{x(r)}(t) = r\, kc_r\, [G(t)]^{r-1} g(t) [1 - G(t)]^{k-r}, \; r = 1,2,3..k \quad (2)$$

If $g(t) = g_{x(1)}(t)$,

In this case it is known that

$$g_{x(1)}(t) = k\, g(t) \left(1 - G(t)\right)^{k-1} \quad (3)$$

By hypothesis $f(t) = \lambda e^{-\lambda t}$ and $g(t) = ce^{-ct}$ (4)

Therefore from (3) and (4) we get,

$$g^*_{x(1)}(\theta) = \frac{kc}{kc + \theta} \quad (5)$$

If $g(t) = g_{x(k)}(t)$,

In this case it is known that

$$g_{x(k)}(t) = k \left(G(t)\right)^{k-1} g(t) \quad (6)$$

Therefore from (4) and (6) we get

$$g^*_{x(k)}(\theta) = \frac{k!\, c^k}{(\theta + c)(\theta + 2c) \dots (\theta + kc)} \quad (7)$$

$$E(T) = -\left.\frac{d(l^*(s))}{ds}\right|_{s=0}, \; E(T^2) = \left.\frac{d^2(l^*(s))}{ds^2}\right|_{s=0} \; and \;\; V(T) = E(T^2) - (E(T))^2 \quad (8)$$

Case (i): The distribution of optional and mandatory thresholds follow exponential distribution.
For this case the first two moments of time to recruitment are found to be
If $g(t) = g_{x(1)}(t)$,

$$E(T) = C_{I1} + C_{I2} - C_{I3} + p\big(C_{I4} + C_{I5} - C_{I6} - H_{I1,4} - H_{I1,5} + H_{I1,6} - H_{I2,4} - H_{I2,5} + H_{I2,6} + H_{I3,4} + H_{I3,5} - H_{I3,6}\big) \quad (9)$$

$$E(T^2) = 2\Big(C_{I1}^2 + C_{I2}^2 - C_{I3}^2 + p\big(C_{I4}^2 + C_{I5}^2 - C_{I6}^2 - H_{I1,4}^2 - H_{I1,5}^2 + H_{I1,6}^2 - H_{I2,4}^2 - H_{I2,5}^2 + H_{I2,6}^2 + H_{I3,4}^2 + H_{I3,5}^2 - H_{I3,6}^2\big)\Big) \quad (10)$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6.

$$C_{Ia} = \frac{1}{\lambda(1 - D_{Ia})} \text{ and } H_{Ib,d} = \frac{1}{\lambda(1 - D_{Ib} D_{Id})} \quad (11)$$

$$D_{I1} = g^*_{x(1)}(\theta_1), D_{I2} = g^*_{x(1)}(\theta_2), D_{I3} = g^*_{x(1)}(\theta_1 + \theta_2)$$
$$D_{I4} = g^*_{x(1)}(\alpha_1), D_{I5} = g^*_{x(1)}(\alpha_2), D_{I6} = g^*_{x(1)}(\alpha_1 + \alpha_2) \text{ are given by (5)} \quad (12)$$

If $g(t) = g_{x(k)}(t)$,

$$E(T) = P_{K1} + P_{K2} - P_{K3} + p\big(P_{K4} + P_{K5} - P_{K6} - Q_{K1,4} - Q_{K1,5} + Q_{K1,6} - Q_{K2,4} - Q_{K2,5} + Q_{K2,6} + Q_{K3,4} + Q_{K3,5} - Q_{K3,6}\big) \quad (13)$$

$$E(T^2) = 2\Big(P_{K1}^2 + P_{K2}^2 - P_{K3}^2 + p\big(P_{K4}^2 + P_{K5}^2 - P_{K6}^2 - Q_{K1,4}^2 - Q_{K1,5}^2 + Q_{K1,6}^2 - Q_{K2,4}^2 - Q_{K2,5}^2 + Q_{K2,6}^2 + Q_{K3,4}^2 + Q_{K3,5}^2 - Q_{K3,6}^2\big)\Big) \quad (14)$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6.

$$P_{Ka} = \frac{1}{\lambda(1 - D_{Ka})} \text{ and } Q_{Kb,d} = \frac{1}{\lambda(1 - D_{Kb} D_{Kd})} \quad (15)$$

$$D_{K1} = g^*_{x(k)}(\theta_1), D_{K2} = g^*_{x(k)}(\theta_2), D_{K3} = g^*_{x(k)}(\theta_1 + \theta_2)$$
$$D_{K4} = g^*_{x(k)}(\alpha_1), D_{K5} = g^*_{x(k)}(\alpha_2), D_{K6} = g^*_{x(k)}(\alpha_1 + \alpha_2) \text{ are given by (7)} \quad (16)$$

The variance of time to recruitment can be calculated from (9), (10), (13) and (14)

Case (ii): The distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = {} & 2C_{I1} + 2C_{I2} + 2C_{I7} + 2C_{I8} - C_{I9} - C_{I10} - C_{I11} - 4C_{I3} + p\big(2C_{I4} + 2C_{I5} + 2C_{I12} + 2C_{I13} - C_{I14} \\ & - C_{I15} - C_{I16} - 4C_{I6} - 4H_{I1,4} - 4H_{I1,5} - 4H_{I1,12} - 4H_{I1,13} + 2H_{I1,14} + 2H_{I1,15} + 2H_{I1,16} + 8H_{I1,6} \\ & - 4H_{I2,4} - 4H_{I2,5} - 4H_{I2,12} - 4H_{I2,13} + 2H_{I2,14} + 2H_{I2,15} + 2H_{I2,16} + 8H_{I2,6} - 4H_{I7,4} - 4H_{I7,5} \\ & - 4H_{I7,12} - 4H_{I7,13} + 2H_{I7,14} + 2H_{I7,15} + 2H_{I7,16} + 8H_{I7,6} - 4H_{I8,4} - 4H_{I8,5} - 4H_{I8,12} - 4H_{I8,13} \\ & + 2H_{I8,14} + 2H_{I8,15} + 2H_{I8,16} + 8H_{I8,6} + 2H_{I9,4} + 2H_{I9,5} + 2H_{I9,12} + 2H_{I9,13} - H_{I9,14} - H_{I9,15} \\ & - H_{I9,16} - 4H_{I9,6} + 2H_{I10,4} + 2H_{I10,5} + 2H_{I10,12} + 2H_{I10,13} - H_{I10,14} - H_{I10,15} - H_{I10,16} - 4H_{I10,6} \\ & + 2H_{I11,4} + 2H_{I11,5} + 2H_{I11,12} + 2H_{I11,13} - H_{I11,14} - H_{I11,15} - H_{I11,16} - 4H_{I11,6} + 8H_{I3,4} + 8H_{I3,5} \\ & + 8H_{I3,12} + 8H_{I3,13} - 4H_{I3,14} - 4H_{I3,15} - 4H_{I3,16} - 16H_{I3,6}\big) \end{aligned} \quad (17)$$

$$\begin{aligned} E(T^2) = 2\Big[& 2C_{I1}^2 + 2C_{I2}^2 + 2C_{I7}^2 + 2C_{I8}^2 - C_{I9}^2 - C_{I10}^2 - C_{I11}^2 - 4C_{I3}^2 + p\big(2C_{I4}^2 + 2C_{I5}^2 + 2C_{I12}^2 + 2C_{I13}^2 - C_{I14}^2 \\ & - C_{I15}^2 - C_{I16}^2 - 4C_{I6}^2 - 4H_{I1,4}^2 - 4H_{I1,5}^2 - 4H_{I1,12}^2 - 4H_{I1,13}^2 + 2H_{I1,14}^2 + 2H_{I1,15}^2 + 2H_{I1,16}^2 + 8H_{I1,6}^2 \\ & - 4H_{I2,4}^2 - 4H_{I2,5}^2 - 4H_{I2,12}^2 - 4H_{I2,13}^2 + 2H_{I2,14}^2 + 2H_{I2,15}^2 + 2H_{I2,16}^2 + 8H_{I2,6}^2 - 4H_{I7,4}^2 - 4H_{I7,5}^2 \\ & - 4H_{I7,12}^2 - 4H_{I7,13}^2 + 2H_{I7,14}^2 + 2H_{I7,15}^2 + 2H_{I7,16}^2 + 8H_{I7,6}^2 - 4H_{I8,4}^2 - 4H_{I8,5}^2 - 4H_{I8,12}^2 - 4H_{I8,13}^2 \\ & + 2H_{I8,14}^2 + 2H_{I8,15}^2 + 2H_{I8,16}^2 + 8H_{I8,6}^2 + 2H_{I9,4}^2 + 2H_{I9,5}^2 + 2H_{I9,12}^2 + 2H_{I9,13}^2 - H_{I9,14}^2 - H_{I9,15}^2 \\ & - H_{I9,16}^2 - 4H_{I9,6}^2 + 2H_{I10,4}^2 + 2H_{I10,5}^2 + 2H_{I10,12}^2 + 2H_{I10,13}^2 - H_{I10,14}^2 - H_{I10,15}^2 - H_{I10,16}^2 - 4H_{I10,6}^2 \\ & + 2H_{I11,4}^2 + 2H_{I11,5}^2 + 2H_{I11,12}^2 + 2H_{I11,13}^2 - H_{I11,14}^2 - H_{I11,15}^2 - H_{I11,16}^2 - 4H_{I11,6}^2 + 8H_{I3,4}^2 + 8H_{I3,5}^2 \\ & + 8H_{I3,12}^2 + 8H_{I3,13}^2 - 4H_{I3,14}^2 - 4H_{I3,15}^2 - 4H_{I3,16}^2 - 16H_{I3,6}^2\big)\Big] \end{aligned} \quad (18)$$

where for a=1, 2, 3…16, b=1, 2, 3,7, 8, 9, 10,11 and d=4, 5, 6, 12, 13,14,15,16.

$C_{Ia}, H_{Ib,d}$ are given by (5) and (11)

$$\begin{aligned} & D_{I7} = g^*_{x(1)}(2\theta_1 + \theta_2), D_{I8} = g^*_{x(1)}(\theta_1 + 2\theta_2), D_{I9} = g^*_{x(1)}(2\theta_1), D_{I10} = g^*_{x(1)}(2\theta_2) \\ & D_{I11} = g^*_{x(1)}(2\theta_1 + 2\theta_2), D_{I12} = g^*_{x(1)}(2\alpha_1 + \alpha_2), D_{I13} = g^*_{x(1)}(2\alpha_2 + \alpha_1), D_{I14} = g^*_{x(1)}(2\alpha_1), \\ & D_{I15} = g^*_{x(1)}(2\alpha_2), D_{I16} = g^*_{x(1)}(2\alpha_1 + 2\alpha_2) \end{aligned} \quad (19)$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned}E(T) = {} & 2P_{K1}+2P_{K2}+2P_{K7}+2P_{K8}-P_{K9}-P_{K10}-P_{K11}-4P_{K3}+p\big(2P_{K4}+2P_{K5}+2P_{K12}+2P_{K13}-P_{K14}\\ & -P_{K15}-P_{K16}-4P_{K6}-4Q_{K1,4}-4Q_{K1,5}-4Q_{K1,12}-4Q_{K1,13}+2Q_{K1,14}+2Q_{K1,15}+2Q_{K1,16}+8Q_{K1,6}\\ & -4Q_{K2,4}-4Q_{K2,5}-4Q_{K2,12}-4Q_{K2,13}+2Q_{K2,14}+2Q_{K2,15}+2Q_{K2,16}+8Q_{K2,6}-4Q_{K7,4}-4Q_{K7,5}\\ & -4Q_{K7,12}-4Q_{K7,13}+2Q_{K7,14}+2Q_{K7,15}+2Q_{K7,16}+8Q_{K7,6}-4Q_{K8,4}-4Q_{K8,5}-4Q_{K8,12}-4Q_{K8,13}\\ & +2Q_{K8,14}+2Q_{K8,15}+2Q_{K8,16}+8Q_{K8,6}+2Q_{K9,4}+2Q_{K9,5}+2Q_{K9,12}+2Q_{K9,13}-Q_{K9,14}-Q_{K9,15}\\ & -Q_{K9,16}-4Q_{K9,6}+2Q_{K10,4}+2Q_{K10,5}+2Q_{K10,12}+2Q_{K10,13}-Q_{K10,14}-Q_{K10,15}-Q_{K10,16}-4Q_{K10,6}\\ & +2Q_{K11,4}+2Q_{K11,5}+2Q_{K11,12}+2Q_{K11,13}-Q_{K11,14}-Q_{K11,15}-Q_{K11,16}-4Q_{K11,6}+8Q_{K3,4}+8Q_{K3,5}\\ & +8Q_{K3,12}+8Q_{K3,13}-4Q_{K3,14}-4Q_{K3,15}-4Q_{K3,16}-16Q_{K3,6}\big)\end{aligned} \tag{20}$$

$$\begin{aligned}E(T^2) = 2\Big[& 2P^2_{K1}+2P^2_{K2}+2P^2_{K7}+2P^2_{K8}-P^2_{K9}-P^2_{K10}-P^2_{K11}-4P^2_{K3}+p\big(2P^2_{K4}+2P^2_{K5}+2P^2_{K12}+2P^2_{K13}-P^2_{K14}\\ & -P^2_{K15}-P^2_{K16}-4P^2_{K6}-4Q^2_{K1,4}-4Q^2_{K1,5}-4Q^2_{K1,12}-4Q^2_{K1,13}+2Q^2_{K1,14}+2Q^2_{K1,15}+2Q^2_{K1,16}+8Q^2_{K1,6}\\ & -4Q^2_{K2,4}-4Q^2_{K2,5}-4Q^2_{K2,12}-4Q^2_{K2,13}+2Q^2_{K2,14}+2Q^2_{K2,15}+2Q^2_{K2,16}+8Q^2_{K2,6}-4Q^2_{K7,4}-4Q^2_{K7,5}\\ & -4Q^2_{K7,12}-4Q^2_{K7,13}+2Q^2_{K7,14}+2Q^2_{K7,15}+2Q^2_{K7,16}+8Q^2_{K7,6}-4Q^2_{K8,4}-4Q^2_{K8,5}-4Q^2_{K8,12}-4Q^2_{K8,13}\\ & +2Q^2_{K8,14}+2Q^2_{K8,15}+2Q^2_{K8,16}+8Q^2_{K8,6}+2Q^2_{K9,4}+2Q^2_{K9,5}+2Q^2_{K9,12}+2Q^2_{K9,13}-Q^2_{K9,14}-Q^2_{K9,15}\\ & -Q^2_{K9,16}-4Q^2_{K9,6}+2Q^2_{K10,4}+2Q^2_{K10,5}+2Q^2_{K10,12}+2Q^2_{K10,13}-Q^2_{K10,14}-Q^2_{K10,15}-Q^2_{K10,16}-4Q^2_{K10,6}\\ & +2Q^2_{K11,4}+2Q^2_{K11,5}+2Q^2_{K11,12}+2Q^2_{K11,13}-Q^2_{K11,14}-Q^2_{K11,15}-Q^2_{K11,16}-4Q^2_{K11,6}+8Q^2_{K3,4}+8Q^2_{K3,5}\\ & +8Q^2_{K3,12}+8Q^2_{K3,13}-4Q^2_{K3,14}-4Q^2_{K3,15}-4Q^2_{K3,16}-16Q^2_{K3,6}\big)\Big]\end{aligned} \tag{21}$$

where for a=1, 2, 3…16, b=1, 2, 3, 7, 8, 9, 10, 11 and d=4, 5, 6, 12, 13,14,15,16.

$P_{Ka}, Q_{Kb,d}$ are given by (7) and (15)

$$\begin{aligned}& D_{K7} = g^*_{x(k)}(2\theta_1+\theta_2), D_{K8} = g^*_{x(k)}(\theta_1+2\theta_2), D_{K9} = g^*_{x(k)}(2\theta_1), D_{K10} = g^*_{x(k)}(2\theta_2), D_{K11} = g^*_{x(k)}(2\theta_1+2\theta_2),\\ & D_{K12} = g^*_{x(k)}(2\alpha_1+\alpha_2), D_{K13} = g^*_{x(k)}(2\alpha_2+\alpha_1), D_{K14} = g^*_{x(k)}(2\alpha_1), D_{K15} = g^*_{x(k)}(2\alpha_2), D_{K16} = g^*_{x(k)}(2\alpha_1+2\alpha_2)\end{aligned} \tag{22}$$

The variance of time to recruitment can be calculated from (17),(18), (20) and (21).

Case (iii): The distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution with shape parameter 2.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned}E(T) = {} & C_{I1}+C_{I2}-C_{I3}+p\big(2C_{I4}+2C_{I5}+2C_{I12}+2C_{I13}-C_{I14}-C_{I15}-C_{I16}-4C_{I6}\\ & -2H_{I1,4}-2H_{I1,5}-2H_{I1,12}-2H_{I1,13}+H_{I1,14}+H_{I1,15}+H_{I1,16}+4H_{I1,6}\\ & -2H_{I2,4}-2H_{I2,5}-2H_{I2,12}-2H_{I2,13}+H_{I2,14}+H_{I2,15}+H_{I2,16}+4H_{I2,6}\\ & +2H_{I3,4}+2H_{I3,5}+2H_{I3,12}+2H_{I3,13}-H_{I3,14}-H_{I3,15}-H_{I3,16}-4H_{I3,6}\big)\end{aligned} \tag{23}$$

$$\begin{aligned}E(T^2) = 2\Big(& C^2_{I1}+C^2_{I2}-C^2_{I3}+p\big(2C^2_{I4}+2C^2_{I5}+2C^2_{I12}+2C^2_{I13}-C^2_{I14}-C^2_{I15}-C^2_{I16}-4C^2_{I6}\\ & -2H^2_{I1,4}-2H^2_{I1,5}-2H^2_{I1,12}-2H^2_{I1,13}+H^2_{I1,14}+H^2_{I1,15}+H^2_{I1,16}+4H^2_{I1,6}\\ & -2H^2_{I2,4}-2H^2_{I2,5}-2H^2_{I2,12}-2H^2_{I2,13}+H^2_{I2,14}+H^2_{I2,15}+H^2_{I2,16}+4H^2_{I2,6}\\ & +2H^2_{I3,4}+2H^2_{I3,5}+2H^2_{I3,12}+2H^2_{I3,13}-H^2_{I3,14}-H^2_{I3,15}-H^2_{I3,16}-4H^2_{I3,6}\big)\Big)\end{aligned} \tag{24}$$

where for a=1,2,3,4,5,6,12,13,14,15,16,b=1,2,3 and d=4,5,6,12,13,14,15,16 $C_{Ia}, H_{Ib,d}$ are given by (5) and (11)

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned}E(T) = {} & P_{K1}+P_{K2}-P_{K3}+p\big(2P_{K4}+2P_{K5}+2P_{K12}+2P_{K13}-P_{K14}-P_{K15}-P_{K16}-4P_{K6}\\ & -2Q_{K1,4}-2Q_{K1,5}-2Q_{K1,12}-2Q_{K1,13}+Q_{K1,14}+Q_{K1,15}+Q_{K1,16}+4Q_{K1,6}\\ & -2Q_{K2,4}-2Q_{K2,5}-2Q_{K2,12}-2Q_{K2,13}+Q_{K2,14}+Q_{K2,15}+Q_{K2,16}+4Q_{K2,6}\\ & +2Q_{K34}+2Q_{K3,5}+2Q_{K3,12}+2Q_{K3,13}-Q_{K3,14}-Q_{K3,15}-Q_{K3,16}-4Q_{K3,6}\big)\end{aligned} \tag{25}$$

$$\begin{aligned}E(T^2) = 2\Big(& P^2_{K1}+P^2_{K2}-P^2_{K3}+p\big(2P^2_{K4}+2P^2_{K5}+2P^2_{K12}+2P^2_{K13}-P^2_{K14}-P^2_{K15}-P^2_{K16}-4P^2_{K6}\\ & -2Q^2_{K1,4}-2Q^2_{K1,5}-2Q^2_{K1,12}-2Q^2_{K1,13}+Q^2_{K1,14}+Q^2_{K1,15}+Q^2_{K1,16}+4Q^2_{K1,6}\\ & -2Q^2_{K2,4}-2Q^2_{K2,5}-2Q^2_{K2,12}-2Q^2_{K2,13}+Q^2_{K2,14}+Q^2_{K2,15}+Q^2_{K2,16}+4Q^2_{K2,6}\\ & +2Q^2_{K3,4}+2Q^2_{K3,5}+2Q^2_{K3,12}+2Q^2_{K3,13}-Q^2_{K3,14}-Q^2_{K3,15}-Q^2_{K3,16}-4Q^2_{K3,6}\big)\Big)\end{aligned} \tag{26}$$

where for a=1,2,3,4,5,6,12,13,14,15,16,b=1,2,3 and d=4,5,6,12,13,14,15,16 $P_{Ka}, Q_{Kb,d}$ are given by (7) and (15)

The variance of time to recruitment can be calculated from (23), (24), (25) and (26).

Case (iv): The distributions of optional and mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned}E(T) = {} & p_2 C_{I1} + q_2 C_{I2} + p_1 C_{I3} - p_1p_2 C_{I4} - p_1q_2 C_{I5} + q_1 C_{I6} - p_2q_1 C_{I7} - q_1q_2 C_{I8} + p\big(p_4 C_{I9} + q_4 C_{I10} + p_3 C_{I11} - p_3p_4 C_{I12} \\ & - p_3q_4 C_{I13} + q_3 C_{I14} - p_4q_3 C_{I15} - q_3q_4 C_{I16} - p_2p_4 H_{I1,9} - p_2q_4 H_{I1,10} - p_2p_3 H_{I1,11} + p_2p_3p_4 H_{I1,12} + p_2p_3q_4 H_{I1,13} \\ & - p_2q_3 H_{I1,14} + p_2p_4q_3 H_{I1,15} + p_2q_3q_4 H_{I1,16} - q_2p_4 H_{I2,9} - q_2q_4 H_{I2,10} - q_2p_3 H_{I2,11} + q_2p_3p_4 H_{I2,12} + q_2p_3q_4 H_{I2,13} \\ & - q_2q_3 H_{I2,14} + q_2p_4q_3 H_{I2,15} + q_2q_3q_4 H_{I2,16} - p_1p_4 H_{I3,9} - p_1q_4 H_{I3,10} - p_1p_3 H_{I3,11} + p_1p_3p_4 H_{I3,12} + p_1p_3q_4 H_{I3,13} - \\ & p_1q_3 H_{I3,14} + p_1p_4q_3 H_{I3,15} + p_1q_3q_4 H_{I3,16} + p_1p_2p_4 H_{I4,9} + p_1p_2q_4 H_{I4,10} + p_1p_2p_3 H_{I4,11} - p_1p_2p_3p_4 H_{I4,12} \\ & - p_1p_2p_3q_4 H_{I4,13} + p_1p_2q_3 H_{I4,14} - p_1p_2p_4q_3 H_{I4,15} - p_1p_2q_3q_4 H_{I4,16} + p_1q_2p_4 H_{I5,9} + p_1q_2q_4 H_{I5,10} + p_1q_2p_3 H_{I5,11} \\ & - p_1q_2p_3p_4 H_{I5,12} - p_1q_2p_3q_4 H_{I5,13} + p_1q_2q_3 H_{I5,14} - p_1q_2p_4q_3 H_{I5,15} - p_1q_2q_3q_4 H_{I5,16} - q_1p_4 H_{I6,9} - q_1q_4 H_{I6,10} \\ & - q_1p_3 H_{I6,11} + q_1p_3p_4 H_{I6,12} + q_1p_3q_4 H_{I6,13} - q_1q_3 H_{I6,14} + q_1p_4q_3 H_{I6,15} + q_1q_3q_4 H_{I6,16} + q_1p_2p_4 H_{I7,9} + q_1p_2q_4 H_{I7,10} \\ & + q_1p_2p_3 H_{I7,11} - q_1p_2p_3p_4 H_{I7,12} - q_1p_2p_3q_4 H_{I7,13} + q_1p_2q_3 H_{I7,14} - q_1p_2p_4q_3 H_{I7,15} - q_1p_2q_3q_4 H_{I7,16} + q_1q_2p_4 H_{I8,9} \\ & + q_1q_2q_4 H_{I8,10} + q_1q_2p_3 H_{I8,11} - q_1q_2p_3p_4 H_{I8,12} - q_1q_2p_3q_4 H_{I8,13} + q_1q_2q_3 H_{I8,14} - q_1q_2p_4q_3 H_{I8,15} - q_1q_2q_3q_4 H_{I8,16}\big)\end{aligned} \tag{27}$$

$$\begin{aligned}E(T^2) = {} & 2\big(p_2 C^2_{I1} + q_2 C^2_{I2} + p_1 C^2_{I3} - p_1p_2 C^2_{I4} - p_1q_2 C^2_{I5} + q_1 C^2_{I6} - p_2q_1 C^2_{I7} - q_1q_2 C^2_{I8} + p\big(p_4 C^2_{I9} + q_4 C^2_{I10} + p_3 C^2_{I11} \\ & - p_3p_4 C^2_{I12} - p_3q_4 C^2_{I13} + q_3 C^2_{I14} - p_4q_3 C^2_{I15} - q_3q_4 C^2_{I16} - p_2p_4 H^2_{I1,9} - p_2q_4 H^2_{I1,10} - p_2p_3 H^2_{I1,11} \\ & + p_2p_3p_4 H^2_{I1,12} + p_2p_3q_4 H^2_{I1,13} - p_2q_3 H^2_{I1,14} + p_2p_4q_3 H^2_{I1,15} + p_2q_3q_4 H^2_{I1,16} - q_2p_4 H^2_{I2,9} \\ & - q_2q_4 H^2_{I2,10} - q_2p_3 H^2_{I2,11} + q_2p_3p_4 H^2_{I2,12} + q_2p_3q_4 H^2_{I2,13} - q_2q_3 H^2_{I2,14} + q_2p_4q_3 H^2_{I2,15} \\ & + q_2q_3q_4 H^2_{I2,16} - p_1p_4 H^2_{I3,9} - p_1q_4 H^2_{I3,10} - p_1p_3 H^2_{I3,11} + p_1p_3p_4 H^2_{I3,12} + p_1p_3q_4 H^2_{I3,13} \\ & - p_1q_3 H^2_{I3,14} + p_1p_4q_3 H^2_{I3,15} + p_1q_3q_4 H^2_{I3,16} + p_1p_2p_4 H^2_{I4,9} + p_1p_2q_4 H^2_{I4,10} + p_1p_2p_3 H^2_{I4,11} \\ & - p_1p_2p_3p_4 H^2_{I4,12} - p_1p_2p_3q_4 H^2_{I4,13} + p_1p_2q_3 H^2_{I4,14} - p_1p_2p_4q_3 H^2_{I4,15} - p_1p_2q_3q_4 H^2_{I4,16} \\ & + p_1q_2p_4 H^2_{I5,9} + p_1q_2q_4 H^2_{I5,10} + p_1q_2p_3 H^2_{I5,11} - p_1q_2p_3p_4 H^2_{I5,12} - p_1q_2p_3q_4 H^2_{I5,13} \\ & + p_1q_2q_3 H^2_{I5,14} - p_1q_2p_4q_3 H^2_{I5,15} - p_1q_2q_3q_4 H^2_{I5,16} - q_1p_4 H^2_{I6,9} - q_1q_4 H^2_{I6,10} - q_1p_3 H^2_{I6,11} \\ & + q_1p_3p_4 H^2_{I6,12} + q_1p_3q_4 H^2_{I6,13} - q_1q_3 H^2_{I6,14} + q_1p_4q_3 H^2_{I6,15} + q_1q_3q_4 H^2_{I6,16} + q_1p_2p_4 H^2_{I7,9} \\ & + q_1p_2q_4 H^2_{I7,10} + q_1p_2p_3 H^2_{I7,11} - q_1p_2p_3p_4 H^2_{I7,12} - q_1p_2p_3q_4 H^2_{I7,13} + q_1p_2q_3 H^2_{I7,14} \\ & - q_1p_2p_4q_3 H^2_{I7,15} - q_1p_2q_3q_4 H^2_{I7,16} + q_1q_2p_4 H^2_{I8,9} + q_1q_2q_4 H^2_{I8,10} + q_1q_2p_3 H^2_{I8,11} \\ & - q_1q_2p_3p_4 H^2_{I8,12} - q_1q_2p_3q_4 H^2_{I8,13} + q_1q_2q_3 H^2_{I8,14} - q_1q_2p_4q_3 H^2_{I8,15} - q_1q_2q_3q_4 H^2_{I8,16}\big)\big)\end{aligned} \tag{28}$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$C_{Ia} = \frac{1}{\lambda(1 - B_{Ia})} \text{ and } H_{Ib,d} = \frac{1}{\lambda(1 - B_{Ib} B_{Id})} \tag{29}$$

$$p_1 = \frac{(\delta_1 - \eta_1)}{(\mu_1 + \delta_1 - \eta_1)}, p_2 = \frac{(\delta_2 - \eta_2)}{(\mu_2 + \delta_2 - \eta_2)}, p_3 = \frac{(\delta_3 - \eta_3)}{(\mu_3 + \delta_3 - \eta_3)}, p_4 = \frac{(\delta_4 - \eta_4)}{(\mu_4 + \delta_4 - \eta_4)}$$

$$q_1 = 1 - p_1, q_2 = 1 - p_2, q_3 = 1 - p_3, q_4 = 1 - p_4$$

where $g^*_{x(1)}(.)$ are given by (5)

$$\begin{aligned} & B_{I1} = g^*_{x(1)}(\delta_2 + \mu_2), B_{I2} = g^*_{x(1)}(\eta_2), B_{I3} = g^*_{x(1)}(\delta_1 + \mu_1), B_{I4} = g^*_{x(1)}(\delta_1 + \mu_1 + \delta_2 + \mu_2), \\ & B_{I5} = g^*_{x(1)}(\delta_1 + \eta_2 + \mu_2), B_{I6} = g^*_{x(1)}(\eta_1), B_{I7} = g^*_{x(1)}(\eta_1 + \delta_2 + \mu_2), B_{I8} = g^*_{x(1)}(\eta_1 + \eta_2), \\ & B_{I9} = g^*_{x(1)}(\delta_4 + \mu_4), B_{I10} = g^*_{x(1)}(\eta_4), B_{I11} = g^*_{x(1)}(\delta_3 + \mu_3), B_{I12} = g^*_{x(1)}(\delta_3 + \mu_3 + \delta_4 + \mu_4), \\ & B_{I13} = g^*_{x(1)}(\delta_3 + \eta_4 + \mu_3), B_{I14} = g^*_{x(1)}(\eta_3), B_{I15} = g^*_{x(1)}(\eta_3 + \delta_4 + \mu_4), B_{I16} = g^*_{x(1)}(\eta_3 + \eta_4)\end{aligned} \tag{30}$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned}
E(T) = {} & p_2 P_{K1} + q_2 P_{K2} + p_1 P_{K3} - p_1 p_2 P_{K4} - p_1 q_2 P_{K5} + q_1 P_{K6} - p_2 q_1 P_{K7} - q_1 q_2 P_{K8} + p\big(p_4 P_{K9} + q_4 P_{K10} + p_3 P_{K11} - p_3 p_4 P_{K12} \\
& - p_3 q_4 P_{K13} + q_3 P_{K14} - p_4 q_3 P_{K15} - q_3 q_4 P_{K16} - p_2 p_4 Q_{K1,9} - p_2 q_4 Q_{K1,10} - p_2 p_3 Q_{K1,11} + p_2 p_3 p_4 Q_{K1,12} + p_2 p_3 q_4 Q_{K1,13} \\
& - p_2 q_3 Q_{K1,14} + p_2 p_4 q_3 Q_{K1,15} + p_2 q_3 q_4 Q_{K1,16} - q_2 p_4 Q_{K2,9} - q_2 q_4 Q_{K2,10} - q_2 p_3 Q_{K2,11} + q_2 p_3 p_4 Q_{K2,12} + q_2 p_3 q_4 Q_{K2,13} \\
& - q_2 q_3 Q_{K2,14} + q_2 p_4 q_3 Q_{K2,15} + q_2 q_3 q_4 Q_{K2,16} - p_1 p_4 Q_{K3,9} - p_1 q_4 Q_{K3,10} - p_1 p_3 Q_{K3,11} + p_1 p_3 p_4 Q_{K3,12} + p_1 p_3 q_4 Q_{K3,13} \\
& - p_1 q_3 Q_{K3,14} + p_1 p_4 q_3 Q_{K3,15} + p_1 q_3 q_4 Q_{K3,16} + p_1 p_2 p_4 Q_{K4,9} + p_1 p_2 q_4 Q_{K4,10} + p_1 p_2 p_3 Q_{K4,11} - p_1 p_2 p_3 p_4 Q_{K4,12} \\
& - p_1 p_2 p_3 q_4 Q_{K4,13} + p_1 p_2 q_3 Q_{K4,14} - p_1 p_2 p_4 q_3 Q_{K4,15} - p_1 p_2 q_3 q_4 Q_{K4,16} + p_1 q_2 p_4 Q_{K5,9} + p_1 q_2 q_4 Q_{K5,10} \\
& + p_1 q_2 p_3 Q_{K5,11} - p_1 q_2 p_3 p_4 Q_{K5,12} - p_1 q_2 p_3 q_4 Q_{K5,13} + p_1 q_2 q_3 Q_{K5,14} - p_1 q_2 p_4 q_3 Q_{K5,15} - p_1 q_2 q_3 q_4 Q_{K5,16} \\
& - q_1 p_4 Q_{K6,9} - q_1 q_4 Q_{K6,10} - q_1 p_3 Q_{K6,11} + q_1 p_3 p_4 Q_{K6,12} + q_1 p_3 q_4 Q_{K6,13} - q_1 q_3 Q_{K6,14} + q_1 p_4 q_3 Q_{K6,15} \\
& + q_1 q_3 q_4 Q_{K6,16} + q_1 p_2 p_4 Q_{K7,9} + q_1 p_2 q_4 Q_{K7,10} + q_1 p_2 p_3 Q_{K7,11} - q_1 p_2 p_3 p_4 Q_{K7,12} - q_1 p_2 p_3 q_4 Q_{K7,13} \\
& + q_1 p_2 q_3 Q_{K7,14} - q_1 p_2 p_4 q_3 Q_{K7,15} - q_1 p_2 q_3 q_4 Q_{K7,16} + q_1 q_2 p_4 Q_{K8,9} + q_1 q_2 q_4 Q_{K8,10} + q_1 q_2 p_3 Q_{K8,11} \\
& - q_1 q_2 p_3 p_4 Q_{K8,12} - q_1 q_2 p_3 q_4 Q_{K8,13} + q_1 q_2 q_3 Q_{K8,14} - q_1 q_2 p_4 q_3 Q_{K8,15} - q_1 q_2 q_3 q_4 Q_{K8,16}\big)
\end{aligned} \tag{31}$$

$$\begin{aligned}
E(T^2) = {} & 2\Big(p_2 P_{K1}^2 + q_2 P_{K2}^2 + p_1 P_{K3}^2 - p_1 p_2 P_{K4}^2 - p_1 q_2 P_{K5}^2 + q_1 P_{K6}^2 - p_2 q_1 P_{K7}^2 - q_1 q_2 P_{K8}^2 + p\big(p_4 P_{K9}^2 + q_4 P_{K10}^2 + p_3 P_{K11}^2 - p_3 p_4 P_{K12}^2 \\
& - p_3 q_4 P_{K13}^2 + q_3 P_{K14}^2 + p_4 q_3 P_{K15}^2 - q_3 q_4 P_{K16}^2 - p_2 p_4 Q_{K1,9}^2 - p_2 q_4 Q_{K1,10}^2 - p_2 p_3 Q_{K1,11}^2 + p_2 p_3 p_4 Q_{K1,12}^2 \\
& + p_2 p_3 q_4 Q_{K1,13}^2 - p_2 q_3 Q_{K1,14}^2 + p_2 p_4 q_3 Q_{K1,15}^2 + p_2 q_3 q_4 Q_{K1,16}^2 - q_2 p_4 Q_{K2,9}^2 - q_2 q_4 Q_{K2,10}^2 - q_2 p_3 Q_{K2,11}^2 \\
& + q_2 p_3 p_4 Q_{K2,12}^2 + q_2 p_3 q_4 Q_{K2,13}^2 - q_2 q_3 Q_{K2,14}^2 + q_2 p_4 q_3 Q_{K2,15}^2 + q_2 q_3 q_4 Q_{K2,16}^2 - p_1 p_4 Q_{K3,9}^2 - p_1 q_4 Q_{K3,10}^2 \\
& - p_1 p_3 Q_{K3,11}^2 + p_1 p_3 p_4 Q_{K3,12}^2 + p_1 p_3 q_4 Q_{K3,13}^2 - p_1 q_3 Q_{K3,14}^2 + p_1 p_4 q_3 Q_{K3,15}^2 + p_1 q_3 q_4 Q_{K3,16}^2 + p_1 p_2 p_4 Q_{K4,9}^2 \\
& + p_1 p_2 q_4 Q_{K4,10}^2 + p_1 p_2 p_3 Q_{K4,11}^2 - p_1 p_2 p_3 p_4 Q_{K4,12}^2 - p_1 p_2 p_3 q_4 Q_{K4,13}^2 + p_1 p_2 q_3 Q_{K4,14}^2 - p_1 p_2 p_4 q_3 Q_{K4,15}^2 \\
& - p_1 p_2 q_3 q_4 Q_{K4,16}^2 + p_1 q_2 p_4 Q_{K5,9}^2 + p_1 q_2 q_4 Q_{K5,10}^2 + p_1 q_2 p_3 Q_{K5,11}^2 - p_1 q_2 p_3 p_4 Q_{K5,12}^2 - p_1 q_2 p_3 q_4 Q_{K5,13}^2 \\
& + p_1 q_2 q_3 Q_{K5,14}^2 - p_1 q_2 p_4 q_3 Q_{K5,15}^2 - p_1 q_2 q_3 q_4 Q_{K5,16}^2 - q_1 p_4 Q_{K6,9}^2 - q_1 q_4 Q_{K6,10}^2 - q_1 p_3 Q_{K6,11}^2 + q_1 p_3 p_4 Q_{K6,12}^2 \\
& + q_1 p_3 q_4 Q_{K6,13}^2 - q_1 q_3 Q_{K6,14}^2 + q_1 p_4 q_3 Q_{K6,15}^2 + q_1 q_3 q_4 Q_{K6,16}^2 + q_1 p_2 p_4 Q_{K7,9}^2 + q_1 p_2 q_4 Q_{K7,10}^2 + q_1 p_2 p_3 Q_{K7,11}^2 \\
& - q_1 p_2 p_3 p_4 Q_{K7,12}^2 - q_1 p_2 p_3 q_4 Q_{K7,13}^2 + q_1 p_2 q_3 Q_{K7,14}^2 - q_1 p_2 p_4 q_3 Q_{K7,15}^2 - q_1 p_2 q_3 q_4 Q_{K7,16}^2 + q_1 q_2 p_4 Q_{K8,9}^2 \\
& + q_1 q_2 q_4 Q_{K8,10}^2 + q_1 q_2 p_3 Q_{K8,11}^2 - q_1 q_2 p_3 p_4 Q_{K8,12}^2 - q_1 q_2 p_3 q_4 Q_{K8,13}^2 + q_1 q_2 q_3 Q_{K8,14}^2 - q_1 q_2 p_4 q_3 Q_{K8,15}^2 - q_1 q_2 q_3 q_4 Q_{K8,16}^2\big)\Big)
\end{aligned} \tag{32}$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$P_{Ka} = \frac{1}{\lambda(1 - B_{Ka})} \text{ and } Q_{Kb,d} = \frac{1}{\lambda(1 - B_{Kb} B_{Kd})} \tag{33}$$

$$\begin{aligned}
& B_{K1} = g^*_{x(k)}(\delta_2 + \mu_2), B_{K2} = g^*_{x(k)}(\eta_2), B_{K3} = g^*_{x(k)}(\delta_1 + \mu_1), B_{K4} = g^*_{x(k)}(\delta_1 + \mu_1 + \delta_2 + \mu_2), \\
& B_{K5} = g^*_{x(k)}(\delta_1 + \eta_2 + \mu_2), B_{K6} = g^*_{x(k)}(\eta_1), B_{K7} = g^*_{x(k)}(\eta_1 + \delta_2 + \mu_2), B_{K8} = g^*_{x(k)}(\eta_1 + \eta_2), \\
& B_{K9} = g^*_{x(k)}(\delta_4 + \mu_4), B_{K10} = g^*_{x(k)}(\eta_4), B_{K11} = g^*_{x(k)}(\delta_3 + \mu_3), B_{K12} = g^*_{x(k)}(\delta_3 + \mu_3 + \delta_4 + \mu_4), \\
& B_{K13} = g^*_{x(k)}(\delta_3 + \eta_4 + \mu_3), B_{K14} = g^*_{x(k)}(\eta_3), B_{K15} = g^*_{x(k)}(\eta_3 + \delta_4 + \mu_4), B_{K16} = g^*_{x(k)}(\eta_3 + \eta_4)
\end{aligned} \tag{34}$$

The variance of time to recruitment can be calculated from (27), (28), (31) and (32).

Case (v): The distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned}
E(T) = {} & C'_{I1} + C'_{I2} - C'_{I3} + p\big(p_4 C_{I9} + q_4 C_{I10} + p_3 C_{I11} - p_3 p_4 C_{I12} - p_3 q_4 C_{I13} + q_3 C_{I14} - p_4 q_3 C_{I15} - q_4 q_3 C_{I16} - \\
& p_4 H'_{I1,9} - q_4 H'_{I1,10} - p_3 H'_{I1,11} + p_3 p_4 H'_{I1,12} + p_3 q_4 H'_{I1,13} - q_3 H'_{I1,14} + p_4 q_3 H'_{I1,15} + q_4 q_3 H'_{I1,16} - \\
& p_4 H'_{I2,9} - q_4 H'_{I2,10} - p_3 H'_{I2,11} + p_3 p_4 H'_{I2,12} + p_3 q_4 H'_{I2,13} - q_3 H'_{I2,14} + p_4 q_3 H'_{I2,15} + q_4 q_3 H'_{I2,16} + \\
& p_4 H'_{I3,9} + q_4 H'_{I3,10} + p_3 H'_{I3,11} - p_3 p_4 H'_{I3,12} - p_3 q_4 H'_{I3,13} + q_3 H'_{I3,14} - p_4 q_3 H'_{I3,15} - q_4 q_3 H'_{I3,16}\big)
\end{aligned} \tag{35}$$

$$\begin{aligned}
E(T^2) = {} & 2\Big(C'^2_{I1} + C'^2_{I2} - C'^2_{I3} + p\big(p_4 C_{I9}{}^2 + q_4 C_{I10}{}^2 + p_3 C_{I11}{}^2 - p_3 p_4 C_{I12}{}^2 - p_3 q_4 C_{I13}{}^2 + q_3 C_{I14}{}^2 - p_4 q_3 C_{I15}{}^2 - q_4 q_3 C_{I16}{}^2 - \\
& p_4 H'^2_{I1,9} - q_4 H'^2_{I1,10} - p_3 H'^2_{I1,11} + p_3 p_4 H'^2_{I1,12} + p_3 q_4 H'^2_{I1,13} - q_3 H'^2_{I1,14} + p_4 q_3 H'^2_{I1,15} + q_4 q_3 H'^2_{I1,16} - \\
& p_4 H'^2_{I2,9} - q_4 H'^2_{I2,10} - p_3 H'^2_{I2,11} + p_3 p_4 H'^2_{I2,12} + p_3 q_4 H'^2_{I2,13} - q_3 H'^2_{I2,14} + p_4 q_3 H'^2_{I2,15} + q_4 q_3 H'^2_{I2,16} + \\
& p_4 H'^2_{I3,9} + q_4 H'^2_{I3,10} + p_3 H'^2_{I3,11} - p_3 p_4 H'^2_{I3,12} - p_3 q_4 H'^2_{I3,13} + q_3 H'^2_{I3,14} - p_4 q_3 H'^2_{I3,15} - q_4 q_3 H'^2_{I3,16}\big)\Big)
\end{aligned} \tag{36}$$

where for b =1, 2,3 and d=9,10,11,12,13,14,15,16.

$$C'_{Ib} = \frac{1}{\lambda(1 - D_{Ib})}, C_{Id} = \frac{1}{\lambda(1 - B_{Id})} \text{ and } H'_{Ib,d} = \frac{1}{\lambda(1 - D_{Ib} B_{Id})} \tag{37}$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned}E(T) = {} & P'_{K1} + P'_{K2} - P'_{K3} + p\big(p_4 P_{K9} + q_4 P_{K10} + p_3 P_{K11} - p_3p_4 P_{K12} - p_3q_4 P_{K13} + q_3 P_{K14} - p_4q_3 P_{K15} - q_4q_3 P_{K16} - \\ & p_4 Q'_{K1,9} - q_4 Q'_{K1,10} - p_3 Q'_{K1,11} + p_3p_4 Q'_{K1,12} + p_3q_4 Q'_{K1,13} - q_3 Q'_{K1,14} + p_4q_3 Q'_{K1,15} + q_4q_3 Q'_{K1,16} - \\ & p_4 Q'_{K2,9} - q_4 Q'_{K2,10} - p_3 Q'_{K2,11} + p_3p_4 Q'_{K2,12} + p_3q_4 Q'_{K2,13} - q_3 Q'_{K2,14} + p_4q_3 Q'_{K2,15} + q_4q_3 Q'_{K2,16} + \\ & p_4 Q'_{K3,9} + q_4 Q'_{K3,10} + p_3 Q'_{K3,11} - p_3p_4 Q'_{K3,12} - p_3q_4 Q'_{K3,13} + q_3 Q'_{K3,14} - p_4q_3 Q'_{K3,15} - q_4q_3 Q'_{K3,16}\big)\end{aligned} \quad (38)$$

$$\begin{aligned}E(T^2) = 2\Big(& P'^2_{K1} + P'^2_{K2} - P'^2_{K3} + p\big(p_4 P_{K9}^2 + q_4 P_{K10}^2 + p_3 P_{K11}^2 - p_3p_4 P_{K12}^2 - p_3q_4 P_{K13}^2 + q_3 P_{K14}^2 - p_4q_3 P_{K15}^2 - q_4q_3 P_{K16}^2 - \\ & p_4 Q'^2_{K1,9} - q_4 Q'^2_{K1,10} - p_3 Q'^2_{K1,11} + p_3p_4 Q'^2_{K1,12} + p_3q_4 Q'^2_{K1,13} - q_3 Q'^2_{K1,14} + p_4q_3 Q'^2_{K1,15} + q_4q_3 Q'^2_{K1,16} - \\ & p_4 Q'^2_{K2,9} - q_4 Q'^2_{K2,10} - p_3 Q'^2_{K2,11} + p_3p_4 Q'^2_{K2,12} + p_3q_4 Q'^2_{K2,13} - q_3 Q'^2_{K2,14} + p_4q_3 Q'^2_{K2,15} + q_4q_3 Q'^2_{K2,16} + \\ & p_4 Q'^2_{K3,9} + q_4 Q'^2_{K3,10} + p_3 Q'^2_{K3,11} - p_3p_4 Q'^2_{K3,12} - p_3q_4 Q'^2_{K3,13} + q_3 Q'^2_{K3,14} - p_4q_3 Q'^2_{K3,15} - q_4q_3 Q'^2_{K3,16}\big)\Big)\end{aligned} \quad (39)$$

where for b =1, 2,3 and d=9,10,11,12,13,14,15,16

$$P'_{Kb} = \frac{1}{\lambda(1-D_{Kb})},\ P_{Kd} = \frac{1}{\lambda(1-B_{Kd})} \text{ and } Q'_{Kb,d} = \frac{1}{\lambda(1-D_{Kb}B_{Kd})} \quad (40)$$

The variance of time to recruitment can be calculated from (35), (36), (38), (39).

Case (vi): The distributions of optional thresholds follow extended exponential distribution with shape parameter 2 and the distribution of mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned}E(T) = {} & 2C'_{I1} + 2C'_{I2} + 2C'_{I7} + 2C'_{I8} - C'_{I9} - C'_{I10} - C'_{I11} - 4C'_{I3} + p\big(p_4 C_{I9} + q_4 C_{I10} + p_3 C_{I11} - p_3p_4 C_{I12} - p_3q_4 C_{I13} \\ & + q_3 C_{I14} - p_4q_3 C_{I15} - q_3q_4 C_{I16} - 2p_4 H'_{I1,9} - 2q_4 H'_{I1,10} - 2p_3 H'_{I1,11} + 2p_3p_4 H'_{I1,12} + 2p_3q_4 H'_{I1,13} \\ & - 2q_3 H'_{I1,14} + 2p_4q_3 H'_{I1,15} + 2q_3q_4 H'_{I1,16} - 2p_4 H'_{I2,9} - 2q_4 H'_{I2,10} - 2p_3 H'_{I2,11} + 2p_3p_4 H'_{I2,12} + 2p_3q_4 H'_{I2,13} \\ & - 2q_3 H'_{I2,14} + 2p_4q_3 H'_{I2,15} + 2q_3q_4 H'_{I2,16} - 2p_4 H'_{I7,9} - 2q_4 H'_{I7,10} - 2p_3 H'_{I7,11} + 2p_3p_4 H'_{I7,12} + 2p_3q_4 H'_{I7,13} \\ & - 2q_3 H'_{I7,14} + 2p_4q_3 H'_{I7,15} + 2q_3q_4 H'_{I7,16} - 2p_4 H'_{I8,9} - 2q_4 H'_{I8,10} - 2p_3 H'_{I8,11} + 2p_3p_4 H'_{I8,12} + 2p_3q_4 H'_{I8,13} \\ & - 2q_3 H'_{I8,14} + 2p_4q_3 H'_{I8,15} + 2q_3q_4 H'_{I8,16} + p_4 H'_{I9,9} + q_4 H'_{I9,10} + p_3 H'_{I9,11} - p_3p_4 H'_{I9,12} - p_3q_4 H'_{I9,13} + q_3 H'_{I9,14} \\ & - p_4q_3 H'_{I9,15} - q_3q_4 H'_{I9,16} + p_4 H'_{I10,9} + q_4 H'_{I10,10} + p_3 H'_{I10,11} - p_3p_4 H'_{I10,12} - p_3q_4 H'_{I10,13} + q_3 H'_{I10,14} - p_4q_3 H'_{I10,15} \\ & - q_3q_4 H'_{I10,16} + p_4 H'_{I11,9} + q_4 H'_{I11,10} + p_3 H'_{I11,11} - p_3p_4 H'_{I11,12} - p_3q_4 H'_{I11,13} + q_3 H'_{I11,14} - p_4q_3 H'_{I11,15} - q_3q_4 H'_{I11,16} + 4p_4 H'_{I13,9} \\ & + 4q_4 H'_{I13,10} + 4p_3 H'_{I13,11} - 4p_3p_4 H'_{I13,12} - 4p_3q_4 H'_{I13,13} + 4q_3 H'_{I13,14} - 4p_4q_3 H'_{I13,15} - 4q_3q_4 H'_{I13,16}\big)\end{aligned} \quad (41)$$

$$\begin{aligned}E(T^2) = 2\Big(& 2C'^2_{I1} + 2C'^2_{I2} + 2C'^2_{I7} + 2C'^2_{I8} - C'^2_{I9} - C'^2_{I10} - C'^2_{I11} - 4C'^2_{I3} + p\big(p_4 C_{I9}^2 + q_4 C_{I10}^2 + p_3 C_{I11}^2 - p_3p_4 C_{I12}^2 - p_3q_4 C_{I13}^2 \\ & + q_3 C_{I14}^2 - p_4q_3 C_{I15}^2 - q_3q_4 C_{I16}^2 - 2p_4 H'^2_{I1,9} - 2q_4 H'^2_{I1,10} - 2p_3 H'^2_{I1,11} + 2p_3p_4 H'^2_{I1,12} + 2p_3q_4 H'^2_{I1,13} \\ & - 2q_3 H'^2_{I1,14} + 2p_4q_3 H'^2_{I1,15} + 2q_3q_4 H'^2_{I1,16} - 2p_4 H'^2_{I2,9} - 2q_4 H'^2_{I2,10} - 2p_3 H'^2_{I2,11} + 2p_3p_4 H'^2_{I2,12} + 2p_3q_4 H'^2_{I2,13} \\ & - 2q_3 H'^2_{I2,14} + 2p_4q_3 H'^2_{I2,15} + 2q_3q_4 H'^2_{I2,16} - 2p_4 H'^2_{I7,9} - 2q_4 H'^2_{I7,10} - 2p_3 H'^2_{I7,11} + 2p_3p_4 H'^2_{I7,12} + 2p_3q_4 H'^2_{I7,13} \\ & - 2q_3 H'^2_{I7,14} + 2p_4q_3 H'^2_{I7,15} + 2q_3q_4 H'^2_{I7,16} - 2p_4 H'^2_{I8,9} - 2q_4 H'^2_{I8,10} - 2p_3 H'^2_{I8,11} + 2p_3p_4 H'^2_{I8,12} + 2p_3q_4 H'^2_{I8,13} \\ & - 2q_3 H'^2_{I8,14} + 2p_4q_3 H'^2_{I8,15} + 2q_3q_4 H'^2_{I8,16} + p_4 H'^2_{I9,9} + q_4 H'^2_{I9,10} + p_3 H'^2_{I9,11} - p_3p_4 H'^2_{I9,12} - p_3q_4 H'^2_{I9,13} + q_3 H'^2_{I9,14} \\ & - p_4q_3 H'^2_{I9,15} - q_3q_4 H'^2_{I9,16} + p_4 H'^2_{I10,9} + q_4 H'^2_{I10,10} + p_3 H'^2_{I10,11} - p_3p_4 H'^2_{I10,12} - p_3q_4 H'^2_{I10,13} + q_3 H'^2_{I10,14} \\ & - p_4q_3 H'^2_{I10,15} - q_3q_4 H'^2_{I10,16} + p_4 H'^2_{I11,9} + q_4 H'^2_{I11,10} + p_3 H'^2_{I11,11} - p_3p_4 H'^2_{I11,12} - p_3q_4 H'^2_{I11,13} + q_3 H'^2_{I11,14} \\ & - p_4q_3 H'^2_{I11,15} - q_3q_4 H'^2_{I11,16} + 4p_4 H'^2_{I13,9} + 4q_4 H'^2_{I13,10} + 4p_3 H'^2_{I13,11} - 4p_3p_4 H'^2_{I13,12} - 4p_3q_4 H'^2_{I13,13} + 4q_3 H'^2_{I13,14} - 4p_4q_3 H'^2_{I13,15} - 4q_3q_4 H'^2_{I13,16}\big)\Big)\end{aligned} \quad (42)$$

where for b =1,2,3,7,8,9,10,11 and d=9,10,11,12,13,14,15,16.

$$C'_{Ib} = \frac{1}{\lambda(1-D_{Ib})},\ C_{Id} = \frac{1}{\lambda(1-B_{Id})} \text{ and } H'_{Ib,d} = \frac{1}{\lambda(1-D_{Ib}B_{Id})} \quad (43)$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = {} & 2P'_{K1} + 2P'_{K2} + 2P'_{K7} + 2P'_{K8} - P'_{K9} - P'_{K10} - P'_{K11} - 4P'_{K3} + p\big(p_4 P_{K9} + q_4 P_{K10} + p_3 P_{K11} - p_3 p_4 P_{K12} - p_3 q_4 P_{K13} \\ & + q_3 P_{K14} - p_4 q_3 P_{K15} - q_3 q_4 P_{K16} - 2p_4 Q'_{K1,9} - 2q_4 Q'_{K1,10} - 2p_3 Q'_{K1,11} + 2p_3 p_4 Q'_{K1,12} + 2p_3 q_4 Q'_{K1,13} \\ & - 2q_3 Q'_{K1,14} + 2p_4 q_3 Q'_{K1,15} + 2q_3 q_4 Q'_{K1,16} - 2p_4 Q'_{K2,9} - 2q_4 Q'_{K2,10} - 2p_3 Q'_{K2,11} + 2p_3 p_4 Q'_{K2,12} + 2p_3 q_4 Q'_{K2,13} \\ & - 2q_3 Q'_{K2,14} + 2p_4 q_3 Q'_{K2,15} + 2q_3 q_4 Q'_{K2,16} - 2p_4 Q'_{K7,9} - 2q_4 Q'_{K7,10} - 2p_3 Q'_{K7,11} + 2p_3 p_4 Q'_{K7,12} + 2p_3 q_4 Q'_{K7,13} \\ & - 2q_3 Q'_{K7,14} + 2p_4 q_3 Q'_{K7,15} + 2q_3 q_4 Q'_{K7,16} - 2p_4 Q'_{K8,9} - 2q_4 Q'_{K8,10} - 2p_3 Q'_{K8,11} + 2p_3 p_4 Q'_{K8,12} + 2p_3 q_4 Q'_{K8,13} \\ & - 2q_3 Q'_{K8,14} + 2p_4 q_3 Q'_{K8,15} + 2q_3 q_4 Q'_{K8,16} + p_4 Q'_{K9,9} + q_4 Q'_{K9,10} + p_3 Q'_{K9,11} - p_3 p_4 Q'_{K9,12} - p_3 q_4 Q'_{K9,13} + q_3 Q'_{K9,14} \\ & - p_4 q_3 Q'_{K9,15} - q_3 q_4 Q'_{K9,16} + p_4 Q'_{K10,9} + q_4 Q'_{K10,10} + p_3 Q'_{K10,11} - p_3 p_4 Q'_{K10,12} - p_3 q_4 Q'_{K10,13} + q_3 Q'_{K10,14} - p_4 q_3 Q'_{K10,15} \\ & - q_3 q_4 Q'_{K10,16} + p_4 Q'_{K11,9} + q_4 Q'_{K11,10} + p_3 Q'_{K11,11} - p_3 p_4 Q'_{K11,12} - p_3 q_4 Q'_{K11,13} + q_3 Q'_{K11,14} - p_4 q_3 Q'_{K11,15} - q_3 q_4 Q'_{K11,16} + 4p_4 Q'_{K13,9} \\ & + 4q_4 Q'_{K3,10} + 4p_3 Q'_{K3,11} - 4p_3 p_4 Q'_{K3,12} - 4p_3 q_4 Q'_{K3,13} + 4q_3 Q'_{K3,14} - 4p_4 q_3 Q'_{K3,15} - 4q_3 q_4 Q'_{K3,16}\big) \end{aligned} \tag{44}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(2P'^{2}_{K1} + 2P'^{2}_{K2} + 2P'^{2}_{K7} + 2P'^{2}_{K8} - P'^{2}_{K9} - P'^{2}_{K10} - P'^{2}_{K11} - 4P'^{2}_{K3} + p\big(p_4 P_{K9}{}^2 + q_4 P_{K10}{}^2 + p_3 P_{K11}{}^2 - p_3 p_4 P_{K12}{}^2 - p_3 q_4 P_{K13}{}^2 \\ & + q_3 P_{K14}{}^2 - p_4 q_3 P_{K15}{}^2 - q_3 q_4 P_{K16}{}^2 - 2p_4 Q'^{2}_{K1,9} - 2q_4 Q'^{2}_{K1,10} - 2p_3 Q'^{2}_{K1,11} + 2p_3 p_4 Q'^{2}_{K1,12} + 2p_3 q_4 Q'^{2}_{K1,13} \\ & - 2q_3 Q'^{2}_{K1,14} + 2p_4 q_3 Q'^{2}_{K1,15} + 2q_3 q_4 Q'^{2}_{K1,16} - 2p_4 Q'^{2}_{K2,9} - 2q_4 Q'^{2}_{K2,10} - 2p_3 Q'^{2}_{K2,11} + 2p_3 p_4 Q'^{2}_{K2,12} + 2p_3 q_4 Q'^{2}_{K2,13} \\ & - 2q_3 Q'^{2}_{K2,14} + 2p_4 q_3 Q'^{2}_{K2,15} + 2q_3 q_4 Q'^{2}_{K2,16} - 2p_4 Q'^{2}_{K7,9} - 2q_4 Q'^{2}_{K7,10} - 2p_3 Q'^{2}_{K7,11} + 2p_3 p_4 Q'^{2}_{K7,12} + 2p_3 q_4 Q'^{2}_{K7,13} \\ & - 2q_3 Q'^{2}_{K7,14} + 2p_4 q_3 Q'^{2}_{K7,15} + 2q_3 q_4 Q'^{2}_{K7,16} - 2p_4 Q'^{2}_{K8,9} - 2q_4 Q'^{2}_{K8,10} - 2p_3 Q'^{2}_{K8,11} + 2p_3 p_4 Q'^{2}_{K8,12} + 2p_3 q_4 Q'^{2}_{K8,13} \\ & - 2q_3 Q'^{2}_{K8,14} + 2p_4 q_3 Q'^{2}_{K8,15} + 2q_3 q_4 Q'^{2}_{K8,16} + p_4 Q'^{2}_{K9,9} + q_4 Q'^{2}_{K9,10} + p_3 Q'^{2}_{K9,11} - p_3 p_4 Q'^{2}_{K9,12} - p_3 q_4 Q'^{2}_{K9,13} + q_3 Q'^{2}_{K9,14} \\ & - p_4 q_3 Q'^{2}_{K9,15} - q_3 q_4 Q'^{2}_{K9,16} + p_4 Q'^{2}_{K10,9} + q_4 Q'^{2}_{K10,10} + p_3 Q'^{2}_{K10,11} - p_3 p_4 Q'^{2}_{K10,12} - p_3 q_4 Q'^{2}_{K10,13} + q_3 Q'^{2}_{K10,14} \\ & - p_4 q_3 Q'^{2}_{K10,15} - q_3 q_4 Q'^{2}_{K10,16} + p_4 Q'^{2}_{K11,9} + q_4 Q'^{2}_{K11,10} + p_3 Q'^{2}_{K11,11} - p_3 p_4 Q'^{2}_{K11,12} - p_3 q_4 Q'^{2}_{K11,13} + q_3 Q'^{2}_{K11,14} \\ & - p_4 q_3 Q'^{2}_{K11,15} - q_3 q_4 Q'^{2}_{K11,16} + 4p_4 Q'^{2}_{K13,9} + 4q_4 Q'^{2}_{K13,10} + 4p_3 Q'^{2}_{K13,11} - 4p_3 p_4 Q'^{2}_{K13,12} - 4p_3 q_4 Q'^{2}_{K13,13} + 4q_3 Q'^{2}_{K13,14} - 4p_4 q_3 Q'^{2}_{K13,15} - 4q_3 q_4 Q'^{2}_{K13,16}\big)\Big) \end{aligned} \tag{45}$$

where for b =1, 2,3,7,8,9,10,11 and d=9,10,11,12,13,14,15,16.

$$P'_{Kb} = \frac{1}{\lambda(1 - D_{Kb})}, \ P_{Kd} = \frac{1}{\lambda(1 - B_{Kd})} \ and \ Q'_{Kb,d} = \frac{1}{\lambda(1 - D_{Kb} B_{Kd})} \tag{46}$$

The variance of time to recruitment can be calculated from (41), (42), (44) and (45).

III. Model Description and Analysis of Model-II

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as Y=min(Y_1, Y_2) and Z=min (Z_1, Z_2). All the other assumptions and notations are as in model-I.

Case (i): The distribution of optional and mandatory thresholds follow exponential distribution

For this case the first two moments of time to recruitment are found to be

Proceeding as in model-I, it can be shown for the present model that

If $g(t) = g_{x(1)}(t)$,

$$E(T) = C_{I3} + p(C_{I6} - H_{I3,6}) \tag{47}$$

$$E(T^2) = 2\left(C^2_{I3} + p(C^2_{I6} - H^2_{I3,6})\right) \tag{48}$$

Where $C_{Ia}, H_{Ib,d}$ are given by equation(5), (11)and (12) for a=3,6, b=3and d=6.

If $g(t) = g_{x(k)}(t)$,

$$E(T) = P_{K3} + p(P_{K6} - Q_{K3,6}) \tag{49}$$

$$E(T^2) = 2\left(P^2_{K3} + p(P^2_{K6} - Q^2_{K3,6})\right) \tag{50}$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15) and (16) for a=3,6, b=3 and d=6.

The variance of time to recruitment can be calculated from (47), (48), (49), (50).

Case (ii): The distribution of optional and mandatory thresholds follow extended exponential distribution

For this case the first two moments of time to recruitment are found to be

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = {} & 4C_{I3} - 2C_{I7} - 2C_{I8} + C_{I11} + p\big(4C_{I6} - 2C_{I12} - 2C_{I13} + C_{I16} - 16H_{I3,6} + 8H_{I3,12} \\ & + 8H_{I3,13} - 4H_{I3,16} + 8H_{I7,6} - 4H_{I7,12} - 4H_{I7,13} + 2H_{I7,6} + 8H_{I8,6} - 4H_{I8,12} - 4H_{I8,13} \\ & + 2H_{I8,16} - 4H_{I11,6} - 2H_{I11,12} + 2H_{I11,13} - H_{I11,16}\big) \end{aligned} \tag{51}$$

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$$E(T^2) = 2\big(4C_{I3}^2 - 2C_{I7}^2 - 2C_{I8}^2 + C_{I11}^2 + p\big(4C_{I6}^2 - 2C_{I12}^2 - 2C_{I13}^2 + C_{I16}^2 - 16H_{I3,6}^2 + 8H_{I3,12}^2 + 8H_{I3,13}^2 - 4H_{I3,16}^2 + 8H_{I7,6}^2 - 4H_{I7,12}^2 - 4H_{I7,13}^2 + 2H_{I7,16}^2 + 8H_{I8,6}^2 - 4H_{I8,12}^2 - 4H_{I8,13}^2 + 2H_{I8,16}^2 - 4H_{I11,6}^2 - 2H_{I11,12}^2 + 2H_{I11,13}^2 - H_{I11,16}^2\big)\big) \quad (52)$$

where $C_{Ia}, H_{Ib,d}$ are given by equation (5), (11), (12) and (19) for a=3, 6,7,8,11,12, 13,16, b=3,7,8,11, and d=6,12,13,16

If $g(t) = g_{x(k)}(t)$,

$$E(T) = 4P_{K3} - 2P_{K7} - 2P_{K8} + P_{K11} + p\big(4P_{K6} - 2P_{K12} - 2P_{K13} + P_{K16} - 16Q_{K3,6} + 8Q_{K3,12} + 8Q_{K3,13} - 4Q_{K3,16} + 8Q_{K7,6} - 4Q_{K7,12} - 4Q_{K7,13} + 2Q_{K7,6} + 8Q_{K8,6} - 4Q_{K8,12} - 4Q_{K8,13} + 2Q_{K8,16} - 4Q_{K11,6} - 2Q_{K11,12} + 2Q_{K11,13} - Q_{K11,16}\big) \quad (53)$$

$$E(T^2) = 2\big(4P_{K3}^2 - 2P_{K7}^2 - 2P_{K8}^2 + P_{K11}^2 + p\big(4P_{K6}^2 - 2P_{K12}^2 - 2P_{K13}^2 + P_{K16}^2 - 16Q_{K3,6}^2 + 8Q_{K3,12}^2 + 8Q_{K3,13}^2 - 4Q_{K3,16}^2 + 8Q_{K7,6}^2 - 4Q_{K7,12}^2 - 4Q_{K7,13}^2 + 2Q_{K7,16}^2 + 8Q_{K8,6}^2 - 4Q_{K8,12}^2 - 4Q_{K8,13}^2 + 2Q_{K8,16}^2 - 4Q_{K11,6}^2 - 2Q_{K11,12}^2 + 2Q_{K11,13}^2 - Q_{K11,16}^2\big)\big) \quad (54)$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15), (16) and (22) for a=3, 6,7,8,11,12, 13,16, b=3,7,8,11, and d=6,12,13,16.

The variance of time to recruitment can be calculated from (51), (52), (53),(54).

Case (iii): The distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution with shape parameter 2.

For this case the first two moments of time to recruitment are found to be

If $g(t) = g_{x(1)}(t)$,

$$E(T) = C_{I3} + p\big(4C_{I6} - 2C_{I12} - 2C_{I3} + C_{I16} - 4H_{I3,6} + 2H_{I3,12} + 2H_{I3,13} - H_{I3,6}\big) \quad (55)$$

$$E(T^2) = 2\big(C_{I3}^2 + p\big(4C_{I6}^2 - 2C_{I12}^2 - 2C_{I13}^2 + C_{I16}^2 - 4H_{I3,6}^2 + 2H_{I3,12}^2 + 2H_{I3,13}^2 - H_{I3,16}^2\big)\big) \quad (56)$$

where $C_{Ia}, H_{Ib,d}$ are given by (5), (11), (12) and (19) for a=3, 6,12, 13,16, b=3 and d=6,12,13,16.

If $g(t) = g_{x(k)}(t)$,

$$E(T) = P_{K3} + p\big(4P_{K6} - 2Q_{K12} - 2Q_{K13} + Q_{K16} - 4Q_{K3,6} + 2Q_{K3,12} + 2Q_{K3,13} - Q_{K3,6}\big) \quad (57)$$

$$E(T^2) = 2\big(P_{K3}^2 + p\big(4P_{K6}^2 - 2P_{K12}^2 - 2P_{K13}^2 + P_{K16}^2 - 4Q_{K3,6}^2 + 2Q_{K3,12}^2 + 2Q_{K3,13}^2 - Q_{K3,16}^2\big)\big) \quad (58)$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15), (16) and (22) for a=3, 6,12, 13,16, b=3 and d=6,12,13,16.

The variance of time to recruitment can be calculated from (55), (56), (57), (58).

Case (iv): The distributions of optional and mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$E(T) = p_1p_2C_{I4} + p_1q_2C_{I5} + p_2q_1C_{I7} + q_1q_2C_{I8} + p\big(p_3p_4C_{I12} + p_3q_4C_{I13} + p_4q_3C_{I15} + q_3q_4C_{I16} - p_1p_2p_3p_4H_{I4,12} - p_1p_2p_3q_4H_{I4,13} - p_1p_2q_3p_4H_{I4,15} - p_1p_2q_3q_4H_{I4,16} - p_1q_2p_3p_4H_{I5,12} - p_1q_2p_3q_4H_{I5,13} - p_1q_2p_4q_3H_{I5,15} - p_1q_2q_3q_4H_{I5,16} - q_1p_2p_3p_4H_{I7,12} - q_1p_2p_3q_4H_{I7,13} - q_1p_2p_4q_3H_{I7,15} - q_1p_2q_3q_4H_{I7,16} - q_1q_2p_3p_4H_{I8,12} - q_1q_2p_3q_4H_{I8,13} - q_1q_2p_4q_3H_{I8,15} - q_1q_2q_3q_4H_{I8,16}\big) \quad (59)$$

$$E(T^2) = 2\big(p_1p_2C_{I4}^2 + p_1q_2C_{I5}^2 + p_2q_1C_{I7}^2 + q_1q_2C_{I8}^2 + p\big(p_3p_4C_{I12}^2 + p_3q_4C_{I13}^2 + p_4q_3C_{I15}^2 + q_3q_4C_{I16}^2 - p_1p_2p_3p_4H_{I4,12}^2 - p_1p_2p_3q_4H_{I4,13}^2 - p_1p_2p_4q_3H_{I4,15}^2 - p_1p_2q_3q_4H_{I4,16}^2 - p_1q_2p_3p_4H_{I5,12}^2 - p_1q_2p_3q_4H_{I5,13}^2 - p_1q_2p_4q_3H_{I5,15}^2 - p_1q_2q_3q_4H_{I5,16}^2 - q_1p_2p_3p_4H_{I7,12}^2 - q_1p_2p_3q_4H_{I7,13}^2 - q_1p_2p_4q_3H_{I7,15}^2 - q_1p_2q_3q_4H_{I7,16}^2 - q_1q_2p_3p_4H_{I8,12}^2 - q_1q_2p_3q_4H_{I8,13}^2 + q_1q_2q_3H_{I8,14}^2 - q_1q_2p_4q_3H_{I8,15}^2 - q_1q_2q_3q_4H_{I8,16}^2\big)\big) \quad (60)$$

where $C_{Ia}, H_{Ib,d}$ are given by (5), (29) and (30) for a=4,5,7,8, 12, 13,15,16, b=4,5,7,8 and d=12, 13,15,16.

If $g(t) = g_{x(k)}(t)$,

$$E(T) = p_1p_2P_{K4} + p_1q_2P_{K5} + p_2q_1P_{K7} + q_1q_2P_{K8} + p\big(p_3p_4P_{K12} + p_3q_4P_{K13} + p_4q_3P_{K15} + q_3q_4P_{K16} - p_1p_2p_3p_4Q_{K4,12} - p_1p_2p_3q_4Q_{K4,13} - p_1p_2q_3p_4Q_{K4,15} - p_1p_2q_3q_4Q_{K4,16} - p_1q_2p_3p_4Q_{K5,12} - p_1q_2p_3q_4Q_{K5,13} - p_1q_2p_4q_3Q_{K5,15} - p_1q_2q_3q_4Q_{K5,16} - q_1p_2p_3p_4Q_{K7,12} - q_1p_2p_3q_4Q_{K7,13} - q_1p_2p_4q_3Q_{K7,15} - q_1p_2q_3q_4Q_{K7,16} - q_1q_2p_3p_4Q_{K8,12} - q_1q_2p_3q_4Q_{K8,13} - q_1q_2p_4q_3Q_{K8,15} - q_1q_2q_3q_4Q_{K8,16}\big) \quad (61)$$

$$E(T^2) = 2\big(p_1p_2P_{K4}^2 + p_1q_2P_{K5}^2 + p_2q_1P_{K7}^2 + q_1q_2P_{K8}^2 + p\big(p_3p_4P_{K12}^2 + p_3q_4P_{K13}^2 + p_4q_3P_{K15}^2 + q_3q_4P_{K16}^2 - p_1p_2p_3p_4Q_{K4,12}^2 - p_1p_2p_3q_4Q_{K4,13}^2 - p_1p_2p_4q_3Q_{K4,15}^2 - p_1p_2q_3q_4Q_{K4,16}^2 - p_1q_2p_3p_4Q_{K5,12}^2 - p_1q_2p_3q_4Q_{K5,13}^2 - p_1q_2p_4q_3Q_{K5,15}^2 - p_1q_2q_3q_4Q_{K5,16}^2 - q_1p_2p_3p_4Q_{K7,12}^2 - q_1p_2p_3q_4Q_{K7,13}^2 - q_1p_2p_4q_3Q_{K7,15}^2 - q_1p_2q_3q_4Q_{K7,16}^2 - q_1q_2p_3p_4Q_{K8,12}^2 - q_1q_2p_3q_4Q_{K8,13}^2 + q_1q_2q_3Q_{K8,14}^2 - q_1q_2p_4q_3Q_{K8,15}^2 - q_1q_2q_3q_4Q_{K8,16}^2\big)\big) \quad (62)$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (33) and (34) for a=4,5,7,8, 12, 13,15,16, b=4,5,7,8 and d=12, 13,15,16.

The variance of time to recruitment can be calculated from (59), (60), (61), (62).

Case (v): If the distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property

If $g(t) = g_{x(1)}(t)$,

$$E(T) = C'_{I3} + p\left(p_3p_4C_{I12} + p_3q_4C_{I13} + p_4q_3C_{I15} + q_3q_4C_{I16} - p_3p_4H'_{I3,12} - p_3q_4H'_{I3,13} - p_4q_3H'_{I3,15} - q_3q_4H'_{I3,16}\right) \tag{63}$$

$$E(T^2) = 2\left(C'^2_{I3} + p\left(p_3p_4{C_{I12}}^2 + p_3q_4{C_{I13}}^2 + p_4q_3{C_{I15}}^2 + q_3q_4{C_{I16}}^2 - p_3p_4H'^2_{I3,12} - p_3q_4H'^2_{I3,13} - p_4q_3H'^2_{I3,15} - q_3q_4H'^2_{I3,16}\right)\right) \tag{64}$$

Where $C_{Id}, C'_{Ib}, H'_{Ib,d}$ are given by equation (5), (11), (12), (29) and (37), b=3 and d=12,13,15,16.

If $g(t) = g_{x(k)}(t)$,

$$E(T) = P'_{K3} + p\left(p_3p_4P_{K12} + p_3q_4P_{K13} + p_4q_3P_{K15} + q_3q_4P_{K16} - p_3p_4Q'_{K3,12} - p_3q_4Q'_{K3,13} - p_4q_3Q'_{K3,15} - q_3q_4Q'_{K3,16}\right) \tag{65}$$

$$E(T^2) = 2\left(P'^2_{K3} + p\left(p_3p_4{P_{K12}}^2 + p_3q_4{Q_{K13}}^2 + p_4q_3{Q_{K15}}^2 + q_3q_4{Q_{K16}}^2 - p_3p_4Q'^2_{K3,12} - p_3q_4Q'^2_{K3,13} - p_4q_3Q'^2_{K3,15} - q_3q_4Q'^2_{K3,16}\right)\right) \tag{66}$$

Where $P_{Kd}, P'_{Kb}, Q'_{Kb,d}$ are given by (7),(15),(16),(33), (34) and (40) for b=3 and d=12,13,15,16.

The variance of time to recruitment can be calculated from (63), (64), (65), and (66).

Case (vi): If the distributions of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$E(T) = 4C'_{I3} - 2C'_{I7} - 2C'_{I8} + C'_{I11} + p\big(p_3p_4C_{I12} - 4p_3p_4 4H'_{I3,12} + 2p_3p_4H'_{I7,12} + 2p_3p_4H'_{I8,12} - p_3p_4H'_{I11,12} \\ q_3p_4C_{I15} - 4q_3p_4H'_{I3,15} + 2q_3p_4H'_{I7,15} + 2q_3p_4H'_{I8,15} - q_3p_4H'_{I11,15} + q_4p_3C_{I13} - 4q_4p_3H'_{I3,13} + 2q_4p_3H'_{I7,13} \\ + 2q_4p_3H'_{I8,13} - q_4p_3H'_{I11,13} + q_3q_4C_{I16} - 4q_3q_4H'_{I3,16} + 2q_3q_4H'_{I7,16} + 2q_3q_4H'_{I8,16} - q_3q_4H'_{I11,16}\big) \tag{67}$$

$$E(T^2) = 2\Big(4C'^2_{I3} - 2C'^2_{I7} - 2C'^2_{I8} + C'^2_{I11} + p\big(p_3p_4{C_{I12}}^2 - 4p_3p_4 4H'^2_{I3,12} + 2p_3p_4H'^2_{I7,12} + 2p_3p_4H'^2_{I8,12} - p_3p_4H'^2_{I11,12} \\ q_3p_4{C_{I15}}^2 - 4q_3p_4H'^2_{I3,15} + 2q_3p_4H'^2_{I7,15} + 2q_3p_4H'^2_{I8,15} - q_3p_4H'^2_{I11,15} + q_4p_3{C_{I13}}^2 - 4q_4p_3H'^2_{I3,13} + 2q_4p_3H'^2_{I7,13} \\ + 2q_4p_3H'^2_{I8,13} - q_4p_3H'^2_{I11,13} + q_3q_4{C_{I16}}^2 - 4q_3q_4H'^2_{I3,16} + 2q_3q_4H'^2_{I17,16} + 2q_3q_4H'^2_{I18,16} - q_3q_4H'^2_{I11,16}\big)\Big) \tag{68}$$

where $C_{Id}, C'_{Ib}, H'_{Ib,d}$ are given by equation (5),(11),(12),(19),(29) and (30), for b=3,7,8,11 and d=12,13,15,16.

If $g(t) = g_{x(k)}(t)$,

$$E(T) = 4P'_{K3} - 2P'_{K7} - 2P'_{K8} + P'_{K11} + p\big(p_3p_4P_{K12} - 4p_3p_4 4Q'_{K3,12} + 2p_3p_4Q'_{K7,12} + 2p_3p_4Q'_{K8,12} - p_3p_4Q'_{K11,12} \\ q_3p_4P_{K15} - 4q_3p_4Q'_{K3,15} + 2q_3p_4Q'_{K7,15} + 2q_3p_4Q'_{K8,15} - q_3p_4Q'_{K11,15} + q_4p_3P_{K13} - 4q_4p_3Q'_{K3,13} + 2q_4p_3Q'_{K7,13} \\ + 2q_4p_3Q'_{K8,13} - q_4p_3Q'_{K11,13} + q_3q_4P_{K16} - 4q_3q_4Q'_{K3,16} + 2q_3q_4Q'_{K7,16} + 2q_3q_4Q'_{K8,16} - q_3q_4Q'_{K11,16}\big) \tag{69}$$

$$E(T^2) = 2\Big(4P'^2_{K3} - 2P'^2_{K7} - 2P'^2_{K8} + P'^2_{K11} + p\big(p_3p_4{P_{K12}}^2 - 4p_3p_4 4Q'^2_{K3,12} + 2p_3p_4Q'^2_{K7,12} + 2p_3p_4Q'^2_{K8,12} - p_3p_4Q'^2_{K11,12} \\ q_3p_4{P_{K15}}^2 - 4q_3p_4Q'^2_{K3,15} + 2q_3p_4Q'^2_{K7,15} + 2q_3p_4Q'^2_{K8,15} - q_3p_4Q'^2_{K11,15} + q_4p_3{P_{K13}}^2 - 4q_4p_3Q'^2_{K3,13} + 2q_4p_3Q'^2_{K7,13} \\ + 2q_4p_3Q'^2_{K8,13} - q_4p_3Q'^2_{K11,13} + q_3q_4{P_{K16}}^2 - 4q_3q_4Q'^2_{K3,16} + 2q_3q_4Q'^2_{K7,16} + 2q_3q_4Q'^2_{K8,16} - q_3q_4Q'^2_{K11,16}\big)\Big) \tag{70}$$

where $P_{Kd}, P'_{Kb}, Q'_{Kb,d}$ are given by (7),(15),(16),(22),(34) and(40) for b=3,7,8,11 d=12,13,15,16

The variance of time to recruitment can be calculated from (67), (68), (69), (70).

IV. MODEL DESCRIPTION AND ANALYSIS OF MODEL-III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y=Y_1+Y_2$ and $Z=Z_1+Z_2$. All the other assumptions and notations are as in model-I. Proceeding as in model-I, it can be shown for the present model that

Case (i): The distributions of optional and mandatory thresholds follow exponential distribution.

If $g(t) = g_{x(1)}(t)$,

$$E(T) = A_2C_{I2} - A_1C_{I1} + p\left(A_5C_{I5} - A_4C_{I4} - A_1A_4H_{I1,4} + A_2A_4H_{I1,4} + A_1A_5H_{I1,5} - A_2A_5H_{I2,5}\right) \tag{71}$$

$$E(T^2) = 2\left(A_2{C_{I2}}^2 - A_1{C_{I1}}^2 + p\left(A_5C^2_{I5} - A_4{C_{I4}}^2 - A_1A_4{H_{I1,4}}^2 + A_2A_4{H_{I1,4}}^2 + A_1A_5{H_{I1,5}}^2 - A_2A_5{H_{I2,5}}^2\right)\right) \tag{72}$$

Where $C_{Ia}, H_{Ib,d}$ are given by equation (5), (11) and (12) for a=1, 2, 4, 5, b=1,2 and d=4, 5

$$A_1 = \frac{\theta 2}{\theta 1 - \theta 2}, A_2 = \frac{\theta 1}{\theta 1 - \theta 2}, A_4 = \frac{\alpha 2}{\alpha 1 - \alpha 2}, A_5 = \frac{\alpha 1}{\alpha 1 - \alpha 2} \tag{73}$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = {} & A_2 P_{K2} - A_1 P_{K1} + p\big(A_5 P_{K5} - A_4 P_{K4} - A_1 A_4 Q_{K1,4} \\ & + A_2 A_4 Q_{K1,4} + A_1 A_5 Q_{K1,5} - A_2 A_5 Q_{K2,5}\big) \end{aligned} \tag{74}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(A_2 {P_{K2}}^2 - A_1 {P_{K1}}^2 + p\big(A_5 P_{K5}^2 - A_4 {P_{K4}}^2 - A_1 A_4 {Q_{K1,4}}^2 \\ & + A_2 A_4 {Q_{K1,4}}^2 + A_1 A_5 {Q_{K1,5}}^2 - A_2 A_5 {Q_{K2,5}}^2\big)\Big) \end{aligned} \tag{75}$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15) and (16) for a=1,2,4,5, b=1,2 and d=4,5

The variance of time to recruitment can be calculated from (71), (72), (74), (75).

Case (ii): If the distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

If $g(t) = g_{x(l)}(t)$,

$$\begin{aligned} E(T) = {} & S_1 C_{I1} - S_2 C_{I9} + S_3 C_{I2} - S_4 C_{I10} + p\big(S_5 C_{I4} - S_6 C_{I14} + S_7 C_{I5} - S_8 C_{I5} \\ & - S_1 S_5 H_{I1,4} + S_1 S_6 H_{I1,14} - S_1 S_7 H_{I1,5} + S_1 S_8 H_{I1,15} \\ & + S_2 S_5 H_{I9,4} - S_2 S_6 H_{I9,14} + S_2 S_7 H_{I9,5} - S_2 S_8 H_{I9,15} \\ & - S_3 S_5 H_{I2,4} + S_3 S_6 H_{I2,14} - S_3 S_7 H_{I2,5} + S_3 S_8 H_{I2,15} \\ & + S_4 S_5 H_{I10,4} - S_4 S_6 H_{I10,14} + S_4 S_7 H_{I10,5} - S_4 S_8 H_{I10,15}\big) \end{aligned} \tag{76}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(S_1 {C_{I1}}^2 - S_2 {C_{I9}}^2 + S_3 {C_{I2}}^2 - S_4 {C_{I10}}^2 + p\big(S_5 {C_{I4}}^2 - S_6 {C_{I14}}^2 + S_7 {C_{I5}}^2 - S_8 {C_{I5}}^2 \\ & - S_1 S_5 {H_{I1,4}}^2 + S_1 S_6 {H_{I1,14}}^2 - S_1 S_7 {H_{I1,5}}^2 + S_1 S_8 {H_{I1,15}}^2 \\ & + S_2 S_5 {H_{I9,4}}^2 - S_2 S_6 {H_{I9,14}}^2 + S_2 S_7 {H_{I9,5}}^2 - S_2 S_8 {H_{I9,15}}^2 \\ & - S_3 S_5 {H_{I2,4}}^2 + S_3 S_6 {H_{I2,14}}^2 - S_3 S_7 {H_{I2,5}}^2 + S_3 S_8 {H_{I2,15}}^2 \\ & + S_4 S_5 {H_{I10,4}}^2 - S_4 S_6 {H_{I10,14}}^2 + S_4 S_7 {H_{I10,5}}^2 - S_4 S_8 {H_{I10,15}}^2\big)\Big) \end{aligned} \tag{77}$$

where $C_{Ia}, H_{Ib,d}$ are given by equation (5), (11), (12) and (19) for a=1, 2,4,5, 9,10,14,15 b=1,2,9,10 and d=4,5,14,15

$$\begin{aligned} & S_1 = \frac{4\theta_2^2}{(\theta_1 - \theta_2)(\theta_1 - 2\theta_2)}, S_2 = \frac{\theta_2^2}{(\theta_1 - \theta_2)(2\theta_1 - \theta_2)}, S_3 = \frac{4\theta_1^2}{(\theta_1 - \theta_2)(2\theta_1 - \theta_2)} \\ & S_4 = \frac{\theta_1^2}{(\theta_1 - \theta_2)(\theta_1 - 2\theta_2)}, S_5 = \frac{4\alpha_2^2}{(\alpha_1 - \alpha_2)(\alpha_1 - 2\alpha_2)}, S_6 = \frac{\alpha_2^2}{(\alpha_1 - \alpha_2)(2\alpha_1 - \alpha_2)} \\ & S_7 = \frac{4\alpha_1^2}{(\alpha_1 - \alpha_2)(2\alpha_1 - \alpha_2)}, S_8 = \frac{\alpha_1^2}{(\alpha_1 - \alpha_2)(\alpha_1 - 2\alpha_2)} \end{aligned} \tag{78}$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = {} & S_1 P_{K1} - S_2 P_{K9} + S_3 P_{K2} - S_4 P_{K10} + p\big(S_5 P_{K4} - S_6 P_{K14} + S_7 P_{K5} - S_8 P_{K5} \\ & - S_1 S_5 Q_{K1,4} + S_1 S_6 Q_{K1,14} - S_1 S_7 Q_{K1,5} + S_1 S_8 Q_{K1,15} + S_2 S_5 Q_{K9,4} - S_2 S_6 Q_{K9,14} \\ & + S_2 S_7 Q_{K9,5} - S_2 S_8 Q_{K9,15} - S_3 S_5 Q_{K2,4} + S_3 S_6 Q_{K2,14} - S_3 S_7 Q_{K2,5} + S_3 S_8 Q_{K2,15} \\ & + S_4 S_5 Q_{K10,4} - S_4 S_6 Q_{K10,14} + S_4 S_7 Q_{K10,5} - S_4 S_8 Q_{K10,15}\big) \end{aligned} \tag{79}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(S_1 {P_{K1}}^2 - S_2 {P_{K9}}^2 + S_3 {P_{K2}}^2 - S_4 {P_{K10}}^2 + p\big(S_5 {P_{K4}}^2 - S_6 {P_{K14}}^2 + S_7 {P_{K5}}^2 - S_8 {P_{K15}}^2 \\ & - S_1 S_5 {Q_{K1,4}}^2 + S_1 S_6 {Q_{K1,14}}^2 - S_1 S_7 {Q_{K1,5}}^2 + S_1 S_8 {Q_{K1,15}}^2 + S_2 S_5 {Q_{K9,4}}^2 - S_2 S_6 {Q_{K9,14}}^2 \\ & + S_2 S_7 {Q_{K9,5}}^2 - S_2 S_8 {Q_{K9,15}}^2 - S_3 S_5 {Q_{K2,4}}^2 + S_3 S_6 {Q_{K2,14}}^2 - S_3 S_7 {Q_{K2,5}}^2 + S_3 S_8 {Q_{K2,15}}^2 \\ & + S_4 S_5 {Q_{K10,4}}^2 - S_4 S_6 {Q_{K10,14}}^2 + S_4 S_7 {Q_{K10,5}}^2 - S_4 S_8 {Q_{K10,15}}^2\big)\Big) \end{aligned} \tag{80}$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15) and (16) for a=1, 2,4,5, 9,10,14,15 b=1,2,9,10 and d=4,5,14,15.

The variance of time to recruitment can be calculated from (76), (77), (79),(80).

Case (iii): If the distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = {} & A_2 C_{I2} - A_1 C_{I1} + p\big(S_5 C_{I4} - S_6 C_{I14} + S_7 C_{I5} - S_8 C_{I15} - A_2 S_5 H_{I2,4} \\ & + A_2 S_6 H_{I2,14} - A_2 S_7 H_{I2,5} + A_2 S_8 H_{I2,15} + A_1 S_5 H_{I1,4} - A_1 S_6 H_{I1,14} \\ & + A_1 S_7 H_{I1,5} - A_1 S_8 H_{I1,15}\big) \end{aligned} \tag{81}$$

$$\begin{aligned} E(T^2) = 2\Big(& A_2 C_{I2}^2 - A_1 C_{I1}^2 + p\big(S_5 C_{I4}^2 - S_6 C_{I14}^2 + S_7 C_{I5}^2 - S_8 C_{I15}^2 - A_2 S_5 H_{I2,4}^2 \\ & + A_2 S_6 H_{I2,14}^2 - A_2 S_7 H_{I2,5}^2 + A_2 S_8 H_{I2,15}^2 + A_1 S_5 H_{I1,4}^2 - A_1 S_6 H_{I1,14}^2 \\ & + A_1 S_7 H_{I1,5}^2 - A_1 S_8 H_{I1,15}^2\big)\Big) \end{aligned} \quad (82)$$

where $C_{Ia}, H_{Ib,d}$ are given by equation (5),(11), (12) and (19) for a=1, 2,4,14,15,5, b=1,2 and d=4,5,14,15

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = & A_2 P_{K2} - A_1 P_{K1} + p\big(S_5 P_{K4} - S_6 P_{K14} + S_7 P_{K5} - S_8 P_{K15} - A_2 S_5 Q_{K2,4} \\ & + A_2 S_6 Q_{K2,14} - A_2 S_7 Q_{K2,5} + A_2 S_8 Q_{K2,15} + A_1 S_5 Q_{K1,4} - A_1 S_6 Q_{K1,14} \\ & + A_1 S_7 Q_{K1,5} - A_1 S_8 Q_{K1,15}\big) \end{aligned} \quad (83)$$

$$\begin{aligned} E(T^2) = 2\Big(& A_2 P_{K2}^2 - A_1 P_{K1}^2 + p\big(S_5 P_{K4}^2 - S_6 P_{K14}^2 + S_7 P_{K5}^2 - S_8 P_{K15}^2 - A_2 S_5 Q_{K2,4}^2 \\ & + A_2 S_6 Q_{K2,14}^2 - A_2 S_7 Q_{K2,5}^2 + A_2 S_8 Q_{K2,15}^2 + A_1 S_5 Q_{K1,4}^2 - A_1 S_6 Q_{K1,14}^2 \\ & + A_1 S_7 Q_{K1,5}^2 - A_1 S_8 Q_{K1,15}^2\big)\Big) \end{aligned} \quad (84)$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7),(15), (16) and (22) for a=1,2,4,5, b=1,2and d=4,5

The variance of time to recruitment can be calculated from (81), (82), (83),(84).

Case (iv): The distributions of optional and the mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = & (R_1 + R_2)C_{I1} - (R_3 + R_4)C_{I3} - (R_5 + R_6)C_{I6} + (R_7 + R_8)C_{I2} + p\big((R_9 + R_{10})C_{I9} + (R_{13} + R_{14})C_{I10} \\ & - (R_{11} + R_{12})C_{I11} - (R_{15} + R_{16})C_{I14} - (R_1 + R_2)\big((R_9 + R_{10})H_{I1,9} + (R_{13} + R_{14})H_{I1,10} \\ & - (R_{11} + R_{12})H_{I1,11} - (R_{15} + R_{16})H_{I1,14}\big) - (R_7 + R_8)\big((R_9 + R_{10})H_{I2,9} + (R_{13} + R_{14})H_{I2,10} \\ & - (R_{11} + R_{12})H_{I2,11} - (R_{15} + R_{16})H_{I2,14}\big) + (R_3 + R_4)\big((R_9 + R_{10})H_{I3,9} + (R_{13} + R_{14})H_{I3,10} \\ & - (R_{11} + R_{12})H_{I3,11} - (R_{15} + R_{16})H_{I3,14}\big) + (R_5 + R_6)\big((R_9 + R_{10})H_{I6,9} + (R_{13} + R_{14})H_{I6,10} \\ & - (R_{11} + R_{12})H_{I6,11} - (R_{15} + R_{16})H_{I6,14}\big)\big) \end{aligned} \quad (85)$$

$$\begin{aligned} E(T^2) = 2\Big(& (R_1 + R_2)C_{I1}^2 - (R_3 + R_4)C_{I3}^2 - (R_5 + R_6)C_{I6}^2 + (R_7 + R_8)C_{I2}^2 + p\big((R_9 + R_{10})C_{I9}^2 + (R_{13} + R_{14})C_{I10}^2 \\ & - (R_{11} + R_{12})C_{I11}^2 - (R_{15} + R_{16})C_{I14}^2 - (R_1 + R_2)\big((R_9 + R_{10})H_{I1,9}^2 + (R_{13} + R_{14})H_{I1,10}^2 \\ & - (R_{11} + R_{12})H_{I1,11}^2 - (R_{15} + R_{16})H_{I1,14}^2\big) - (R_7 + R_8)\big((R_9 + R_{10})H_{I2,9}^2 + (R_{13} + R_{14})H_{I2,10}^2 \\ & - (R_{11} + R_{12})H_{I2,11}^2 - (R_{15} + R_{16})H_{I2,14}^2\big) + (R_3 + R_4)\big((R_9 + R_{10})H_{I3,9}^2 + (R_{13} + R_{14})H_{I3,10}^2 \\ & - (R_{11} + R_{12})H_{I3,11}^2 - (R_{15} + R_{16})H_{I3,14}^2\big) + (R_5 + R_6)\big((R_9 + R_{10})H_{I6,9}^2 + (R_{13} + R_{14})H_{I6,10}^2 \\ & - (R_{11} + R_{12})H_{I6,11}^2 - (R_{15} + R_{16})H_{I6,14}^2\big)\big)\Big) \end{aligned} \quad (86)$$

where $C_{Ia}, H_{Ib,d}$ are given by equation (5), (11), (29) and (30) for a=1,3,6,9,10,11,12,14, b=1,2,3,6 and d=9,10,11,14

$$\begin{aligned} & R_1 = \frac{(\delta_1 + \mu_1)p_1 p_2}{(\delta_1 - \delta_2 + \mu_1 - \mu_2)}, R_2 = \frac{\eta_1 q_1 p_2}{(\eta_1 - \delta_2 - \mu_2)}, R_3 = \frac{(\delta_2 + \mu_2)p_1 p_2}{(\delta_1 - \delta_2 + \mu_1 - \mu_2)}, R_4 = \frac{\eta_2 p_1 q_2}{(\delta_1 - \eta_2 + \mu_1)} R_5 = \frac{(\delta_2 + \mu_2)q_1 p_2}{(\eta_1 - \delta_2 - \mu_2)}, \\ & R_6 = \frac{\eta_2 q_1 q_2}{(\eta_1 - \eta_2)}, R_7 = \frac{(\delta_1 + \mu_1)p_1 q_2}{(\delta_1 + \mu_1 - \eta_2)}, R_8 = \frac{\eta_1 q_1 q_2}{(\eta_1 - \eta_2)} R_9 = \frac{(\delta_3 + \mu_3)p_3 p_4}{(\delta_3 - \delta_4 + \mu_3 - \mu_4)}, R_{10} = \frac{\eta_3 q_3 p_4}{(\eta_3 - \delta_4 - \mu_4)}, \\ & R_{11} = \frac{(\delta_4 + \mu_4)p_3 p_4}{(\delta_3 - \delta_4 + \mu_3 - \mu_4)}, R_{12} = \frac{\eta_4 p_3 q_4}{(\delta_3 - \eta_4 + \mu_3)} R_{13} = \frac{(\delta_3 + \mu_3)q_4 p_3}{(\delta_3 - \eta_4 + \mu_3)}, R_{14} = \frac{\eta_3 q_3 q_4}{(\eta_3 - \eta_4)}, R_{15} = \frac{(\delta_4 + \mu_4)p_4 q_3}{(\eta_3 - \delta_4 - \mu_4)}, R_{16} = \frac{\eta_4 q_3 q_4}{(\eta_3 - \eta_4)} \end{aligned} \quad (87)$$

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = & (R_1 + R_2)P_{K1} - (R_3 + R_4)P_{K3} - (R_5 + R_6)P_{K6} + (R_7 + R_8)P_{K2} + p\big((R_9 + R_{10})P_{K9} + (R_{13} + R_{14})P_{K10} \\ & - (R_{11} + R_{12})P_{K11} - (R_{15} + R_{16})P_{K14} - (R_1 + R_2)\big((R_9 + R_{10})Q_{K1,9} + (R_{13} + R_{14})Q_{K1,10} \\ & - (R_{11} + R_{12})Q_{K1,11} - (R_{15} + R_{16})Q_{K1,14}\big) - (R_7 + R_8)\big((R_9 + R_{10})Q_{K2,9} + (R_{13} + R_{14})Q_{K2,10} \\ & - (R_{11} + R_{12})Q_{K2,11} - (R_{15} + R_{16})Q_{K2,14}\big) + (R_3 + R_4)\big((R_9 + R_{10})Q_{K3,9} + (R_{13} + R_{14})Q_{K3,10} \\ & - (R_{11} + R_{12})Q_{K3,11} - (R_{15} + R_{16})Q_{K3,14}\big) + (R_5 + R_6)\big((R_9 + R_{10})Q_{K6,9} + (R_{13} + R_{14})Q_{K6,10} \\ & - (R_{11} + R_{12})Q_{K6,11} - (R_{15} + R_{16})Q_{K6,14}\big)\big) \end{aligned} \quad (88)$$

$$\begin{aligned} E(T^2) = 2\Big(& (R_1 + R_2)P_{K1}^2 - (R_3 + R_4)P_{K3}^2 - (R_5 + R_6)P_{K6}^2 + (R_7 + R_8)P_{K2}^2 + p\big((R_9 + R_{10})P_{K9}^2 + (R_{13} + R_{14})P_{K10}^2 \\ & - (R_{11} + R_{12})P_{K11}^2 - (R_{15} + R_{16})P_{K14}^2 - (R_1 + R_2)\big((R_9 + R_{10})Q_{K1,9}^2 + (R_{13} + R_{14})Q_{K1,10}^2 \\ & - (R_{11} + R_{12})Q_{K1,11}^2 - (R_{15} + R_{16})Q_{K1,14}^2\big) - (R_7 + R_8)\big((R_9 + R_{10})Q_{K2,9}^2 + (R_{13} + R_{14})Q_{K2,10}^2 \\ & - (R_{11} + R_{12})Q_{K2,11}^2 - (R_{15} + R_{16})Q_{K2,14}^2\big) + (R_3 + R_4)\big((R_9 + R_{10})Q_{K3,9}^2 + (R_{13} + R_{14})Q_{K3,10}^2 \\ & - (R_{11} + R_{12})Q_{K3,11}^2 - (R_{15} + R_{16})Q_{K3,14}^2\big) + (R_5 + R_6)\big((R_9 + R_{10})Q_{K6,9}^2 + (R_{13} + R_{14})Q_{K6,10}^2 \\ & - (R_{11} + R_{12})Q_{K6,11}^2 - (R_{15} + R_{16})Q_{K6,14}^2\big)\big)\Big) \end{aligned} \quad (89)$$

where $P_{Ka}, Q_{Kb,d}$ are given by (7), (15), (33)and (34) for a=1,3,6,9,10,11,12,14, b=1,2,3,6 and d=9,10,11,14

The variance of time to recruitment can be calculated from (85), (86), (88), (89).

Case (v): The distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = {} & A_2 C'_{I2} - A_1 C'_{I1} + p\big[(R_9+R_{10})C_{I9} + (R_{13}+R_{14})C_{I10} - (R_{11}+R_{12})C_{I11} - (R_{15}+R_{16})C_{I14} - A_2(R_9+R_{10})H'_{I2,9} - \\ & A_2(R_{13}+R_{14})H'_{I2,10} + A_2(R_{11}+R_{12})H'_{I2,11} + A_2(R_{15}+R_{16})H'_{I2,14} + A_1(R_9+R_{10})H'_{I1,9} + \\ & A_1(R_{13}+R_{14})H'_{I1,10} - A_1(R_{11}+R_{12})H'_{I1,11} - A_1(R_{15}+R_{16})H'_{I1,14}\big] \end{aligned} \tag{90}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(A_2 C'^2_{I2} - A_1 C'^2_{I1} + p\big[(R_9+R_{10})C_{I9}{}^2 + (R_{13}+R_{14})C_{I10}{}^2 - (R_{11}+R_{12})C_{I11}{}^2 - (R_{15}+R_{16})C_{I14}{}^2 - A_2(R_9+R_{10})H'^2_{I2,9} \\ & - A_2(R_{13}+R_{14})H'^2_{I2,10} + A_2(R_{11}+R_{12})H'^2_{I2,11} + A_2(R_{15}+R_{16})H'^2_{I2,14} + A_1(R_9+R_{10})H'^2_{I1,9} + \\ & A_1(R_{13}+R_{14})H'^2_{I1,10} - A_1(R_{11}+R_{12})H'^2_{I1,11} - A_1(R_{15}+R_{16})H'^2_{I1,14}\big]\Big) \end{aligned} \tag{91}$$

where $C_{Id}, C'_{Ib}, H_{Ib,d}$ are given by equation (5), (11), (12), (29) and (30), for b=1,2 and d=9,10,11,14.

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = {} & A_2 P'_{K2} - A_1 P'_{K1} + p\big[(R_9+R_{10})P_{K9} + (R_{13}+R_{14})P_{K10} - (R_{11}+R_{12})P_{K11} - (R_{15}+R_{16})P_{K14} - A_2(R_9+R_{10})Q'_{K2,9} - \\ & A_2(R_{13}+R_{14})Q'_{K2,10} + A_2(R_{11}+R_{12})Q'_{K2,11} + A_2(R_{15}+R_{16})Q'_{K2,14} + A_1(R_9+R_{10})Q'_{K1,9} + \\ & A_1(R_{13}+R_{14})Q'_{K1,10} - A_1(R_{11}+R_{12})Q'_{K1,11} - A_1(R_{15}+R_{16})Q'_{K1,14}\big] \end{aligned} \tag{92}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(A_2 P'^2_{K2} - A_1 P'^2_{K1} + p\big[(R_9+R_{10})P_{K9}{}^2 + (R_{13}+R_{14})P_{K10}{}^2 - (R_{11}+R_{12})P_{K11}{}^2 - (R_{15}+R_{16})P_{K14}{}^2 \\ & - A_2(R_9+R_{10})Q'^2_{K2,9} - A_2(R_{13}+R_{14})Q'^2_{K2,10} + A_2(R_{11}+R_{12})Q'^2_{K2,11} + A_2(R_{15}+R_{16})Q'^2_{K2,14} + A_1(R_9+R_{10})Q'^2_{K1,9} + \\ & A_1(R_{13}+R_{14})Q'^2_{K1,10} - A_1(R_{11}+R_{12})Q'^2_{K1,11} - A_1(R_{15}+R_{16})Q'^2_{K1,14}\big]\Big) \end{aligned} \tag{92}$$

where $P_{Id}, P'_{Ib}, Q_{Ib,d}$ are given by (7), (15), (16), (33) and (34) for b=1,2, a=9,10,11,14 and d=12,13,15,16.

The variance of time to recruitment can be calculated from (90), (91), (92) and (93).

Case (vi): If the distributions of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t)$,

$$\begin{aligned} E(T) = {} & S_1 C'_{I1} + S_3 C'_{I2} - S_4 C'_{I10} - S_2 C'_{I9} + p\big((R_9+R_{10})C_{I9} + (R_{13}+R_{14})C_{I10} - (R_{11}+R_{12})C_{I11} - (R_{15}+R_{16})C_{I14} \\ & - S_1(R_9+R_{10})H'_{I1,9} - S_1(R_{13}+R_{14})H'_{I1,10} + S_1(R_{11}+R_{12})H'_{I1,11} + S_1(R_{15}+R_{16})H'_{I1,14} - S_3(R_9+R_{10})H'_{I2,9} \\ & - S_3(R_{13}+R_{14})H'_{I2,10} + S_3(R_{11}+R_{12})H'_{I2,11} + S_3(R_{15}+R_{16})H'_{I2,14} + S_4(R_9+R_{10})H'_{I10,9} + S_4(R_{13}+R_{14})H'_{I10,10} \\ & - S_4(R_{11}+R_{12})H'_{I10,11} - S_4(R_{15}+R_{16})H'_{I10,14} + S_2(R_9+R_{10})H'_{I9,9} + S_2(R_{13}+R_{14})H'_{I9,10} - S_2(R_{11}+R_{12})H'_{I9,11} - S_2(R_{15}+R_{16})H'_{I9,14}\big) \end{aligned} \tag{94}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(S_1 C'^2_{I1} + S_3 C'^2_{I2} - S_4 C'^2_{I10} - S_2 C'^2_{I9} + p\big((R_9+R_{10})C^2_{I9} + (R_{13}+R_{14})C^2_{I10} - (R_{11}+R_{12})C^2_{I11} - (R_{15}+R_{16})C^2_{I14} \\ & - S_1(R_9+R_{10})H'^2_{I1,9} - S_1(R_{13}+R_{14})H'^2_{I1,10} + S_1(R_{11}+R_{12})H'^2_{I1,11} + S_1(R_{15}+R_{16})H'^2_{I1,14} - S_3(R_9+R_{10})H'^2_{I2,9} \\ & - S_3(R_{13}+R_{14})H'^2_{I2,10} + S_3(R_{11}+R_{12})H'^2_{I2,11} + S_3(R_{15}+R_{16})H'^2_{I2,14} + S_4(R_9+R_{10})H'^2_{I10,9} + S_4(R_{13}+R_{14})H'^2_{I10,10} \\ & - S_4(R_{11}+R_{12})H'^2_{I10,11} - S_4(R_{15}+R_{16})H'^2_{I10,14} + S_2(R_9+R_{10})H'^2_{I9,9} + S_2(R_{13}+R_{14})H'^2_{I9,10} - S_2(R_{11}+R_{12})H'^2_{I9,11} - S_2(R_{15}+R_{16})H'^2_{I9,14}\big)\Big) \end{aligned} \tag{95}$$

where

$$S_1 = \frac{4\theta_2^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)}, S_1 = \frac{\theta_2^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)}, S_3 = \frac{4\theta_1^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)}, \text{and } S_4 = \frac{\theta_1^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)} \tag{96}$$

where $C_{Id}, C'_{Ib}, H_{Ib,d}$ are given by equation (5), (11), (12), (19), (29) and (30), for b=1,2,9,10, d=9,10,11,14

If $g(t) = g_{x(k)}(t)$,

$$\begin{aligned} E(T) = {} & S_1 P'_{K1} + S_3 P'_{K2} - S_4 P'_{K10} - S_2 P'_{K9} + p\big((R_9+R_{10})P_{K9} + (R_{13}+R_{14})P_{K10} - (R_{11}+R_{12})P_{K11} - (R_{15}+R_{16})P_{K14} \\ & - S_1(R_9+R_{10})Q'_{K1,9} - S_1(R_{13}+R_{14})Q'_{K1,10} + S_1(R_{11}+R_{12})Q'_{K1,11} + S_1(R_{15}+R_{16})Q'_{K1,14} - S_3(R_9+R_{10})Q'_{K2,9} \\ & - S_3(R_{13}+R_{14})Q'_{K2,10} + S_3(R_{11}+R_{12})Q'_{K2,11} + S_3(R_{15}+R_{16})Q'_{K2,14} + S_4(R_9+R_{10})Q'_{K10,9} + S_4(R_{13}+R_{14})Q'_{K10,10} \\ & - S_4(R_{11}+R_{12})Q'_{K10,11} - S_4(R_{15}+R_{16})Q'_{K10,14} + S_2(R_9+R_{10})Q'_{K9,9} + S_2(R_{13}+R_{14})Q'_{K9,10} - S_2(R_{11}+R_{12})Q'_{K9,11} - S_2(R_{15}+R_{16})Q'_{K9,14}\big) \end{aligned} \tag{97}$$

$$\begin{aligned} E(T^2) = {} & 2\Big(S_1 P'^2_{K1} + S_3 P'^2_{K2} - S_4 P'^2_{K10} - S_2 P'^2_{K9} + p\big((R_9+R_{10})P'^2_{K9} + (R_{13}+R_{14})P'^2_{K10} - (R_{11}+R_{12})P'^2_{K11} - (R_{15}+R_{16})P'^2_{K14} \\ & - S_1(R_9+R_{10})Q'^2_{K1,9} - S_1(R_{13}+R_{14})Q'^2_{K1,10} + S_1(R_{11}+R_{12})Q'^2_{K1,11} + S_1(R_{15}+R_{16})Q'^2_{K1,14} - S_3(R_9+R_{10})Q'^2_{K2,9} \\ & - S_3(R_{13}+R_{14})Q'^2_{K2,10} + S_3(R_{11}+R_{12})Q'^2_{K2,11} + S_3(R_{15}+R_{16})Q'^2_{K2,14} + S_4(R_9+R_{10})Q'^2_{K10,9} + S_4(R_{13}+R_{14})Q'^2_{K10,10} \\ & - S_4(R_{11}+R_{12})Q'^2_{K10,11} - S_4(R_{15}+R_{16})Q'^2_{K10,14} + S_2(R_9+R_{10})Q'^2_{K9,9} + S_2(R_{13}+R_{14})Q'^2_{K9,10} - S_2(R_{11}+R_{12})Q'^2_{K9,11} - S_2(R_{15}+R_{16})Q'^2_{K9,14}\big)\Big) \end{aligned} \tag{98}$$

where $P_{Kd}, P'_{Kb}, Q_{Kb,d}$ are given by (7), (15), (16), (22), (34), (40) for b=1,2,9,10, d=9,10,11,14.

The variance of time to recruitment can be calculated from (94), (95), (97), (98).

V. NUMERICAL ILLUSTRATIONS

The mean and variance of the time to recruitment for the above models are given in the following tables for the cases (i),(ii),(iii),(iv),(v),(vi) respectively by keeping $\theta_1 = 0.4, \theta_2 = 0.6, \alpha_1 = 0.5, \alpha_2 = 0.8, P = 0.8.$ $\delta_1 = 0.6, \eta_1 = 0.3, \mu_1 = 0.7, \delta_2 = 0.4, \eta_2 = 0.7, \mu_2 = 0.4, \delta_3 = 0.5, \eta_3 = 0.2, \mu_3 = 0.5, \delta_4 = 0.8, \eta_4 = 0.4, \mu_4 = 0.2$ fixed and varying c, k, λ one at a time and the results are tabulated below.

Table 1: Effect of c, k, λ on Performance Measures

MODEL-I									
K	1	2	3	2	2	2	2	2	2
C	1.5	1.5	1.5	0.5	1	1.5	1.5	1.5	1.5
λ	1	1	1	1	1	1	0.6	0.7	0.8
E(T) (n=1)	6.9679	12.7114	18.4451	5.0451	8.8847	12.7114	24.2139	20.1059	17.0248
V(T) (n=1)	25.6665	77.4020	156.5814	14.5131	39.8637	77.4020	153.1010	127.7187	107.4767
E(T) (n=k)	6.9679	4.730	3.9241	2.1596	3.4497	4.73	9.1033	7.5414	6.37
V(T) (n=k)	25.6665	12.6869	9.1335	3.5011	7.4237	12.6869	27.8966	22.5103	18.4372
E(T) (n=1)	7.7324	14.1669	20.6183	5.5940	9.8742	14.1669	29.7872	24.2085	20.0245
V(T) (n=1)	43.6104	144.2556	302.2485	22.9082	70.7631	144.2556	257.3404	227.0730	196.9932
E(T) (n=k)	7.7324	5.3442	4.4931	2.4912	3.9306	5.3442	11.0848	9.0346	7.4970
V(T) (n=k)	43.6104	19.7909	13.4981	4.3699	10.5594	19.7909	40.9110	34.3086	28.6710
E(T) (n=1)	7.6598	14.0574	20.4440	5.5180	9.7949	14.0574	27.1538	22.4765	18.9686
V(T) (n=1)	27.3582	81.5901	164.0955	15.5572	42.2954	81.5901	144.2872	126.6968	110.0029
E(T) (n=k)	7.6598	5.1876	4.297	2.3247	3.7619	5.1876	10.0894	8.3388	7.0258
V(T) (n=k)	27.3582	13.5404	9.7527	3.7578	7.9558	13.5404	28.1981	23.3401	19.4218
E(T) (n=1)	6.7761	12.3669	17.9494	4.9055	8.6416	12.3669	23.0643	19.2438	16.3784
V(T) (n=1)	29.2323	92.6577	191.6421	15.9851	46.4246	92.6577	200.5393	160.3362	131.4045
E(T) (n=k)	6.7761	3.9552	3.1877	1.8868	2.9243	3.9552	7.5765	6.2832	5.3132
V(T) (n=k)	29.2323	9.8094	6.5992	2.8943	5.8699	9.8094	22.3054	17.7392	14.3899
E(T) (n=1)	6.3514	11.5099	16.6596	4.6242	8.0730	11.5099	21.6086	18.0019	15.2969
V(T) (n=1)	23.1721	69.8898	141.5116	13.1266	35.9789	69.8898	144.1815	117.6021	97.7016
E(T) (n=k)	6.3514	4.3220	3.6021	2.013	3.1717	4.3220	8.2299	6.8342	5.7874
V(T) (n=k)	23.1721	11.4928	8.2743	3.2180	6.7497	11.4928	25.7557	20.5339	16.7108
E(T) (n=1)	7.9834	14.7965	21.6066	5.7088	10.2552	14.7965	27.4189	22.9109	19.5299
V(T) (n=1)	28.7849	86.5084	174.4565	16.2611	44.6674	86.5084	159.6305	135.3133	115.7645
E(T) (n=k)	7.9834	7.8489	4.4903	4.4390	4.4913	4.5322	5.8079	5.3523	5.0106
V(T) (n=k)	28.7849	28.7427	10.2917	10.1313	10.1873	10.4479	11.9987	10.9724	10.5471

Table 2: Effect of c, k, λ on Performance Measures

MODEL-II									
K	1	2	3	2	2	2	2	2	2
C	1.5	1.5	1.5	0.5	1	1.5	1.5	1.5	1.5
λ	1	1	1	1	1	1	0.6	0.7	0.8
E(T) (n=1)	3.0441	4.9681	6.8787	2.3932	3.6884	4.9681	9.3989	7.8165	6.6296
V(T) (n=1)	7.1810	17.7836	33.0360	4.6687	10.2009	17.7836	42.1363	32.8947	26.3603
E(T) (n=k)	3.0441	2.1339	1.8138	1.1612	1.4750	1.8138	3.6202	2.9750	2.4912
V(T) (n=k)	7.1810	3.8254	2.8885	1.3288	2.0241	2.8885	7.6222	5.8456	4.5759
E(T) (n=1)	4.4497	7.6880	10.9106	3.3580	5.5327	7.6880	14.7677	12.2392	10.3429
V(T) (n=1)	10.6673	26.6871	49.4034	6.8065	15.2649	26.6871	52.1370	43.9696	37.1718
E(T) (n=k)	4.4497	3.0517	2.5507	1.5918	2.3263	3.0517	5.9568	4.9193	4.1411
V(T) (n=k)	10.6673	5.6193	4.2102	2.1447	3.7298	5.6193	13.0870	10.4268	8.4267
E(T) (n=1)	3.5702	5.9807	8.3777	2.7560	4.3768	5.9807	11.4176	9.4759	8.0196
V(T) (n=1)	8.4758	21.0017	38.8392	5.4729	12.0623	21.0017	46.9688	37.5616	30.6033
E(T) (n=k)	3.5702	2.4771	2.0880	1.4025	1.9395	2.4771	4.8561	4.0065	3.3692
V(T) (n=k)	8.4758	4.486	3.3744	1.8087	3.0247	4.486	11.1229	8.6697	6.8903
E(T) (n=1)	2.9380	4.7725	6.5941	2.3177	3.5523	4.7725	8.9574	7.4628	6.3498
V(T) (n=1)	7.1321	18.0254	33.8971	4.5951	10.2126	18.0254	43.1655	33.4669	26.7180
E(T) (n=k)	2.9380	2.0662	1.7357	1.2719	1.6632	2.0662	4.0552	3.3449	2.8121
V(T) (n=k)	7.1321	3.7882	2.8182	1.5791	2.5610	3.7882	9.6770	7.4454	5.8653
E(T) (n=1)	2.8512	4.5971	6.3298	2.2599	3.4360	4.5971	8.6575	7.2073	6.1198
V(T) (n=1)	6.5790	16.1355	29.8839	4.3129	9.3013	16.1355	38.4553	29.8989	23.9082
E(T) (n=k)	2.8512	2.0081	1.6877	1.2535	1.6248	2.0081	3.9560	3.2603	2.7386
V(T) (n=k)	6.5790	3.5292	2.6356	1.5321	2.4273	3.5292	9.0636	6.9736	5.4889
E(T) (n=1)	3.9369	6.7623	9.5806	2.9886	4.8804	6.7623	12.4654	10.4286	8.9009
V(T) (n=1)	9.8711	25.3047	41.959	6.2277	14.2632	25.3047	55.6542	44.2466	36.1055
E(T) (n=k)	3.9369	2.7209	2.2689	1.4672	2.0916	2.7209	5.1947	4.3112	3.6485
V(T) (n=k)	9.8711	5.1899	3.8730	1.9539	3.4137	5.1899	12.3144	9.6421	7.7311

Table 3: Effect of c, k, λ on Performance Measures

MODEL-III									
K	1	2	3	2	2	2	2	2	2
C	1.5	1.5	1.5	0.5	1	1.5	1.5	1.5	1.5
λ	1	1	1	1	1	1	0.6	0.7	0.8
E(T) (n=1)	8.7025	16.1601	23.6093	6.2092	11.1903	16.1601	30.8934	25.6315	21.6851
V(T) (n=1)	34.8211	107.5901	219.6924	19.3	54.7082	107.5901	193.7135	168.0582	145.0958
E(T) (n=k)	8.7025	5.8844	4.8678	2.5533	4.2237	5.8844	11.3321	9.3865	7.9273
V(T) (n=k)	34.8211	17.0293	12.1710	4.3730	9.7346	17.0293	35.0322	28.9995	24.1856
E(T) (n=1)	11.9437	22.5534	33.1521	8.3979	15.4828	22.5534	44.0534	36.3748	30.6159
V(T) (n=1)	44.2200	133.7518	270.1036	24.7717	68.8635	133.7518	123.3951	151.6207	154.9058
E(T) (n=k)	11.9437	8.0357	6.6236	3.3	5.6738	8.0357	15.7359	12.9858	10.9233
V(T) (n=k)	44.2200	21.7335	15.5694	5.6084	12.5201	21.7335	32.5263	31.1588	28.3197
E(T) (n=1)	9.8360	18.3833	26.9212	6.9787	12.6873	18.3833	35.7166	29.5262	24.8833
V(T) (n=1)	37.8434	115.3035	233.7771	21.1329	59.1073	115.3035	171.4821	163.3494	148.5502
E(T) (n=k)	9.836	6.6357	5.4806	2.8196	4.7327	6.6357	12.9483	10.6938	9.0029
V(T) (n=k)	37.8434	18.5379	13.2601	4.8008	10.66	18.5379	34.6404	30.0184	25.7287
E(T) (n=1)	8.4755	15.7514	23.0206	6.0443	10.9024	15.7514	29.4338	24.5472	20.8823
V(T) (n=1)	40.082	129.9102	270.8655	21.4945	64.3452	129.9102	264.8733	216.4212	180.2652
E(T) (n=k)	8.4755	4.3769	3.3948	2.0278	3.2051	4.3769	8.4063	6.9672	5.8879
V(T) (n=k)	40.0820	11.6592	7.4287	3.2325	6.8214	11.6592	26.2674	20.9829	17.0686
E(T) (n=1)	7.9598	14.7112	21.4554	5.7029	10.2120	14.7112	27.6702	23.0420	19.5708
V(T) (n=1)	32.0076	99.3772	203.4804	17.7082	50.3838	99.3772	190.9037	159.9947	135.5157
E(T) (n=k)	7.9598	5.3930	4.4673	2.3770	3.8894	5.3930	10.2507	8.5158	7.2146
V(T) (n=k)	32.0076	15.6775	11.2170	4.0428	8.9588	15.6775	33.2772	27.0374	22.3241
E(T) (n=1)	10.6990	20.2346	29.7701	7.5192	13.8777	20.2346	37.3603	31.2440	26.6568
V(T) (n=1)	43.2219	133.8285	273.0144	23.8208	68.0251	133.8285	216.0017	192.6595	170.7740
E(T) (n=k)	10.6990	7.2243	5.9683	2.9798	5.1042	7.2243	13.4696	11.2391	9.5663
V(T) (n=k)	43.2219	21.0947	15.0517	5.2428	11.9675	21.0947	39.5573	33.4406	28.5052

VI. Findings

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment for all the models are reported below.

i. It is observed that if k increases, the mean and variance of the time to recruitment of all the models increase when the probability density function of loss of man-hours is probability density function of first order statistics and decreases when it is probability density function of k-th order statistics.
ii. If c increases, the average number of exits increases, which, in turn, implies that mean and variance of the time to recruitment increase for all the models.
iii. As λ increases, the average inter-decision time decreases, which, in turn, shows that frequent decisions are made on the average and hence mean and variance of the time to recruitment decrease for all the models.

VII. Conclusion

Note that while the time to recruitment is postponed in model-III, the time to recruitment is advanced in model-I and II. Therefore from the organization point of view, model III is more preferable.

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